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Optimal Placement of Phasor Measurement Units in Electric Power System Networks- A Review

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ABSTRACT

The contribution of Phasor Measurement Units (PMUs) to modern electrical power systems by providing real-time, synchronized measurements of electrical parameters across the grid for monitoring, stability and reliability of power system cannot be overstated. However, due to the high costs associated with PMU installation and maintenance, an optimal placement strategy is essential to achieve maximum system coverage using minimal resources. This problem is formulated as the optimal PMU placement (OPP) problem with the goal of minimizing the number of PMUs required to ensure observability of the power system. This paper offers a comprehensive review of the OPP problem, with a focus on both single- and multi-objective formulations of the problem, the practical constraints as well as a variety of algorithms and techniques used in solving the problem. This review offers researchers the information required to select the most appropriate and practical computational approaches for their specific applications. The paper also highlights promising directions for future research, including dynamic network reconfiguration, integration of cybersecurity measures, and the incorporation of PMU placement strategies within microgrids.

INTRODUCTION

To ensure the security of power systems, realtime monitoring is critical for maintaining situational awareness of operating conditions. In recent years, the power sector has faced major challenges as consumers demand not only conventional and renewable energy sources but also competitive pricing, reliable service, and protection against outages. To address these challenges, increasing attention has been directed toward the deployment of Phasor Measurement Units (PMUs) for the protection, monitoring, and control of modern power systems.

PMUs provide GPS-synchronized, real-time voltage measurements, making them highly effective in enhancing fault location and overall system observability. This capability gives PMUs a significant advantage over traditional Supervisory Control and Data Acquisition (SCADA) systems. With advanced digital features, PMUs have become indispensable in present and future power system operations. Their ability to support real-time data collection, monitoring, and storage ensures comprehensive visibility of system performance, even under rapidly changing operating conditions (Jha et al, 2022).

Despite these advantages, the high costs associated with procuring, installing, and commissioning PMUs necessitate the development of optimal placement strategies to minimize the number required while ensuring full network observability. This challenge is typically formulated as an optimization problem known as Optimal PMU Placement (OPP), where the objective is often to minimize the number or cost of units installed. The literature presents various approaches to solving the traditional OPP problem. Some studies extend the formulation to include additional objectives such as maximizing system observability and redundancy, or minimizing communication infrastructure costs, thereby framing the problem as a multiobjective OPP (Palmero et al., 2021). The OPP is usually treated as a constrained optimization problem, incorporating conditions such as zeroinjection buses (ZIBs), budget limits, and PMU channel restrictions. This review highlights the different formulations of the OPP problem, focusing on commonly adopted objective functions, constraints, and the solution techniques in the literature.

FORMULATION OF OPTIMAL PMU PLACEMENT PROBLEM

The objective of optimal PMU placement problem is to determine the PMU placement locations in such a way to minimize the number of PMUs required to ensure complete observability of the power system network.

A. Basic OPP formulation to minimize the number of PMUs

Given a power network with N buses, the bus connectivity matrix defined as 1, $if\ bus\ i\ and\ j\ are\ connected$

$$A_{ij} = \begin{cases} 1, & i = j \\ 0, & otherwise \end{cases} \qquad (1)$$

and $x_i \in \{0,1\}$ which is a binary variable defined such that

$$x_i = \begin{cases} 1, & if\ a\ PMU\ is\ placed\ at\ bus\ i \\ 0, & otherwise \end{cases} \qquad (2)$$

the OPP problem, in its most basic form, is typically formulated mathematically as

$$f_i = min \sum_{i=1}^{N} x_i \qquad (3)$$

where the goal is to minimize the number of PMUs subject to an observability constraint

$$\sum_{j=1}^{N} A_{ij} x_j \geq 1 \quad \forall i = 1, \dots, N \qquad (4)$$

The topological observability constraint ensures that the voltage phasor of each bus is measurable, either directly by a PMU or indirectly through Kirchhoff's laws applied to adjacent buses. This ensures each bus is observed at least once (Nuqui and Phadke, 2005).

B. Alternative Objective Functions in the Literature

Apart from the basic OPP that minimizes the number of PMUs, other objective functions that have been incorporated into the problem formulation. These include:

(i) Minimization of Installation Cost

If the cost of installation c_i of a PMU unit varies per bus as a result of geographical or infrastructure differences, the installation cost is minimized using (Cruz et al., 2019):

$$f_2 = min \sum_{i=1}^{N} c_i x_i \qquad (5)$$

(ii) Maximization of System Observability Redundancy Index (SORI)

SORI is a measure of redundancy in the number of times a particular bus is observed by PMUs (Okendo et al., 2021). If the total number of PMUs (including the neighbours) that can observe bus i is given by

$$y_i = \sum_{j=1}^{N} A_{ij} x_j \qquad (6)$$

Then the objective function for maximization of SORI is given by (Arul et al., 2015):

$$f_3 = max \sum_{i=1}^{N} y_i \qquad (7)$$

The objective seeks to improve the robustness of observability, especially under contingencies such as PMU or line failures.

(iii) Maximization of Measurement Redundancy for Critical Buses

In some systems, only certain buses, e.g. buses to which generators or substations are connected are considered as critical. Let $C \subset \{1, \dots, N\}$ be the set of critical buses. Then

$$f_4 = max \sum_{i \in C}^{N} y_i \qquad (8)$$

prioritizes PMU placement to ensure the observability of critical infrastructure over the rest of the network. Esmaili and GhamsariYazdel (2017) proposed a channel-oriented method for installing PMU to monitor fragile areas to prevent voltage collapse.

(iv) Maximization of Topological Observability Index (TOI)

Topological Observability Index accounts for observability under probabilistic failure scenarios. If p_j represents the probability that the PMU at bus j is functional, then the objective to maximize the probability that bus i is observable is given by (Huang et al, 2019):

$$f_5 = max \sum_{i=1}^{N} \left(1 - \prod_{J=1}^{N} \left(1 - p_j A_{ij} x_j \right) \right) \qquad (9)$$

(v) Minimization of cyber-vulnerability risk This objective incorporates cyber-physical considerations for secure placement of PMUs. Let v_i represent the vulnerability score of a PMU placed at bus i. Then the objective function is (Khare et al., 2021):

$$f_6 = min \sum_{i=1}^{N} v_i x_i \qquad (10)$$

(vi) Minimization of Voltage Stability Index This objective requires that PMUs should be placed at a bus in order to monitor the critical lines having a higher value of VSI such that such vulnerable bus should be monitored by at least 2 PMUs. The objective function is formulated as (Haggi et al., 2020):

$$f_6 = min \sum_{ij=1}^{N} VSI_{ij} x_j \ \forall i, \qquad J = 1, \dots, N \quad (11a)$$

$$2 \leq voi \leq m + 1 \qquad (11b)$$

where VSI_{ij} is the voltage stability index with maximum value of unity, voi is the voltage stability observability index and m is the number of branches connected to each bus.

C. Other PMU Placement Constraints Apart from the full observability constraint given in equation (4), various constraints are considered in the Optimal PMU placement problem depending on the model, observability level and system requirements. These include:

Zero Injection Bus (ZIB) constraint A ZIB is a bus that has neither a generator or a load connected to it. A key advantage of ZIBs in PMU placement arises from Kirchhoff's Current Law, which permits the voltage phasors of adjacent buses to be inferred without direct measurement by PMUs (Baldwin et al., 1993). This results in a reduction in the PMU count and the costs associated with PMU hardware and communication infrastructure. Let $Z \subset \{1, \dots, N\}$ be the set of ZIBs. For a ZIB at bus i, if all but one of its neighbouring buses are observable, then the unobservable one can be inferred by KCL. ZIB constraint is often encoded using additional binary variables or constraint programming techniques (Milosevic and Begovic, 2003).

Redundancy Constraint (k-observability)
Redundancy constraints in PMU placement are introduced to ensure power system observability during contingencies such as PMU failures, communication disruptions, or transmission line outages by establishing backup observability paths, ensuring that critical state estimation tasks can proceed without interruption (Xu and Abur, 2004; Kamis and Salama, 2011; Aghaei et al., 2015). To ensure that each bus is observed by at least k PMUs (Li et al., 2011),

$$\sum_{j=1}^{N} A_{ij} x_j \geq 1 \quad \forall i = 1, \ldots, N \tag{12}$$

The constraint increases PMU requirements but improves system resilience.

PMU Channel Limit Constraint The total number of measurements taken by a PMU cannot exceed its total channel capacity. This constraint can be expressed as (Philip and Jain, 2018):

$$\sum_{i=1}^{N} a_{ij} w_{ij} \leq w_j^{max} \quad \forall i \in I \tag{13}$$

where w_{ij} denotes whether the bus i is observed using a PMU placed at bus j, w_j^{max} is the channel capacity of the PMU placed at j^{th}bus and I is the set of buses in the power system.

Budget Constraint
Budget constraints reflect the financial limitations of utilities regarding equipment procurement, installation, communication infrastructure, and maintenance (Nuqui & Phadke, 2005). In real-world scenarios, it is rarely feasible to install PMUs on every bus due to high costs, making it necessary to optimize placement within a predefined budget. If the total budget for PMU installation is B, and the cost per PMU is c_i, then

$$\sum_{i=1}^{N} c_i x_i \leq B \tag{14}$$

Placement Constraint Set If PMUs cannot be installed at certain buses, for example due to infrastructure constraints, then (Khodadadi-Arpanahi et al. 2023):

$$x_i = 0 \; \forall i \in F \tag{15}$$

where F is the set of buses where PMU installation is forbidden.

Phasing Constraint Phasing installation constraint is required when, for economic reasons, there is a limited number of PMUs to be installed in the power system at various time stages. It is given by (Esmaili and Ghamsari-Yazde, 2017):

$$\sum_{j=1}^{N} x_i \leq TN_{PMU} \tag{16}$$

Communication Constraints in PMU Placement

Communication constraints refer to the practical limitations and requirements of phasor data being transmitted from PMUs to control centers or Phasor Data Concentrators (PDCs) (Khan et al., 2012). These constraints significantly influence optimal PMU placement decisions as the infrastructure needed for reliable and low-latency data transmission can be costly or geographically limited. Key aspects of communication constraints in PMUs placement are bandwidth limitations for realtime transmission, latency and delay (Zhang et al., 2010, Chen et al., 2017).

D. Multi-objective Optimal PMU Placement

Practical applications often require balancing multiple objectives. Multi-objective optimization of simultaneously optimizing two or more conflicting objectives subject to a set of constraints. For the OPP problem, when other objective functions are considered simultaneously with the minimization of the number of PMUs, a multi-objective optimal PMU placement problem results. In multiobjective optimization, a set of trade-off solutions, known as non-dominated or Paretooptimal solutions, where no objective can be improved without degrading the others is produced.

Classical methods of solving multi-objective optimization problems include the weighted sum (Marler & Arora, 2004), ε-constraint method and goal programming approaches (Deb, 2001). In the weighted sum method, the objectives are scalarized by forming their weighted sum while in the ε-constraint method, a most preferred objective is optimized and the others are restricted within user-specified values. The goal programming methods requires the decision maker to set desired goals for each objective, and the optimization aims to minimize deviations from these goals.

Another class of methods for solving multiobjective optimization problems is classified as evolutionary and metaheuristic algorithms. These nature-based methods work with a population of solutions. Therefore, they do not require the objective functions to be scalarized and they are well-suited for exploring the Pareto front.

METHODS FOR OPTIMAL PMU PLACEMENT

Researchers have developed various methods to address the OPP problem, broadly classified into mathematical programming and heuristic algorithms.

A. Mathematical Programming Mathematical programming methods are deterministic methods that leverage rigorous mathematical frameworks to provide globally optimal or near-optimal solutions. Examples include integer linear programming (ILP), mixed integer linear programming (MILP), mixed integer non-linear programming (MINLP) and Semidefinite Programming

(SDP). Mathematical programming approaches to PMU placement formulate the OPP as an optimization problem with integer constraints. The most common representation involves binary decision variables indicating PMU presence at each bus. The objective is typically to minimize the total number of PMUs while maintaining full system observability. The basic ILP formulation minimizes the summation of binary variables corresponding to buses, subject to observability constraints represented in matrix form. These constraints ensure that either a PMU is installed on a bus or that the bus is adjacent to another with a PMU. The standard ILP model can be extended into Mixed Integer Linear Programming (MILP) to accommodate additional system constraints, such as zero injection buses (ZIBs), budget limitations, and measurement channel capacities. In the case of ZIBs, mathematical formulations exploit the Kirchhoff's Current Law (KCL) to deduce the observability of unmonitored buses, thereby reducing the number of required PMUs.

Nonlinear programming (NLP) formulations help to handle scenarios where system observability criteria or the objective functions are nonlinear. For example, when minimizing state estimation error becomes the goal rather than the number of PMUs, the objective becomes nonlinear, and the resulting optimization problem cannot be solved using traditional linear programming techniques. Although NLP methods offer greater flexibility, they often converge to local optima, especially in non-convex landscapes, which reduces their reliability for large systems unless supplemented with global search strategies or sensitivity analysis.

Semidefinite programming (SDP) is a powerful subclass of convex optimization methods (Ahmed et al., 2020). It enables the reformulation of inherently discrete and combinatorial problems traditionally modeled with binary decision variables into continuous optimal PMU placement (OPP) problems that can be efficiently addressed using modern interior-point algorithms (Wang et al., 2009). This is achieved by relaxing the binary placement variables and expressing the problem within a semidefinite matrix framework, thereby making it computationally tractable while preserving acceptable solution quality.

Nuqui and Phadke (2005) proposed a binary decision model to minimize the number of Phasor Measurement Units to achieve full system observability. Their work was limited to single-objective optimization and small systems due to computational complexity. Gou (2008) used branch-and-bound algorithms to solve the ILP formulation for IEEE test systems up to 118 buses. Korkali and Abur (2012) also solved the OPP problem by employing MILP solvers like CPLEX, improving efficiency but still facing scalability limitations for large systems with contingency constraints. Xu and Abur (2004) applied ILP to OPP, solving for IEEE 14-bus, 57-bus and 118-bus systems. The study focused on single-objective minimization without addressing redundancy or contingencies.

Binary linear programming approaches for optimal PMU location considering power system state estimation and various contingencies were presented by Enshaee et al (2012). Theodorakatos et al. (2014) implemented the NLP based sequential quadratic programming to obtain near-optimal solutions for such nonlinear problems. Shaaban et al. (2019) proposed a multi-objective OPP for power quality monitoring devices to minimize cost and loss of observability via branch-and-reduce-based non-linear programming global solver that minimizes conflicting objectives. Abdolahi et al. (2021) proposed reliability-based ILP schemes that integrate μ-PMUs into the distribution grid while accounting for various system contingencies while Yuce et al. (2022) presented a new distribution-level phasor measurement unit (D-PMU) placement to reconcile reliability of high-precision observation. Al-Hinai et al. (2023) introduced the fault location observability (FLO) concept to minimize PMU installation cost and maximize observability based on the location of fault in the system and employed MILP method to solve the OPP problem using GAMS and CPLEX solvers.

Kekatos et al (2012) solved the optimal placement of Phasor Measurement Units via convex relaxation. Li et al (2013) formulated the placement problem as an integerconstrained SDP, and employed relaxations and randomization methods to approximate optimal solutions. Manousakis et al (2015) developed a binary SDP (BSDP) model to minimize the number of PMUs while ensuring numerical observability, explicitly incorporating zeroinjection buses and conventional measurements. Manousakis and Korres (2015) introduced channel capacity limits into the BSDP framework, ensuring that PMUs cannot measure more branches than their hardware allows. These studies highlight how SDP-based relaxations can yield near-optimal solutions to placement problems that are otherwise computationally difficult. Peng et al (2023) developed a MISDP model for optimal microPMU placement in distribution systems, with the objective of improving state estimation accuracy under the E-optimal criterion.

B. Heuristic Algorithms

Genetic Algorithm

The genetic algorithm (GA) approach is developed from the principle of natural selection, and notably does not require the computation of derivatives. GA offers several advantages that make it particularly suitable for solving optimization problems, including a reduced possibility of being caught up in local minima, elimination of need to calculate transitions between states, and fitness evaluation guides the search processes of each candidate outcome (or string) (Lee & El-Sharkawi, 2008).

Milosevic and Begovic (2003) proposed a theoretical graph-based approach combined with Genetic Algorithms (GA) to derive Paretooptimal solutions using the Non-dominated Sorting Genetic Algorithm (NSGA). Their method was applied to determine the minimum number of Phasor Measurement Units (PMUs) required for effective system monitoring, addressing two conflicting objectives: minimizing the number of PMUs and maximizing measurement redundancy in the OPP solution.

Similarly, Marin et al. (2003) introduced a GAbased strategy for solving the Optimal PMU Placement problem, aimed at achieving complete system observability in linear state estimation, thereby providing valuable insights for efficient PMU allocation. Vedik and Shiva (2020) solved the multi-objective OPP problem considering optimizing PMU installation cost and maximization of measurement redundancy simultaneously via non-dominated sorting genetic algorithm-II (NSGA-II)

Particle Swarm Optimization

Similar to GAs, Particle Swarm Optimization (PSO) begins with a population of randomly generated solutions. In PSO, these solutions called particles move within a multidimensional search space. Their movements are sequential, and particle velocities are restricted to ensure they remain within valid regions and do not exceed predefined bounds (Eberhart and Kennedy, 1995).

Hajian et al. (2011) introduced a modified Binary Particle Swarm Optimization (BPSO) approach to solve the Optimal PMU Placement (OPP) problem. Their method was designed to achieve complete system observability while considering practical constraints such as PMU failures and transmission line outages. Furthermore, Hajian et al. (2011) integrated control reconfigurability into their formulation, developing a fault-tolerant PMU placement strategy that optimizes installation while treating the number of available PMUs as a key constraint. Ahmadi et al. (2011) utilized a rectified version of BPSO to enhance optimization performance in solving the OPP problem to obtain complete system observability.

Simulated Annealing

Simulated Annealing (SA) is a metaheuristic optimization technique designed to solve complex combinatorial problems by introducing random perturbations to the current solution. Effective performance on large-scale problems depends on carefully defining the perturbation strategy, cost function, solution space, and cooling schedule. According to Gou (2008), SA is particularly effective for large systems and exhibits efficient convergence toward optimal or near-optimal solutions.

Simulated annealing has been applied to the OPP problem, particularly in scenarios where communication constraints limit full observability (Nuqui and Phadke, 2005). To address challenges related to bad data detection, Kerdchuen and Ongsakul (2007) introduced a topological observability-based method to enhance OPP effectiveness. In a related effort, Kerdchuen and Ongsakul, 2008 implemented a stochastic extension of Simulated Annealing which they called Stochastic Simulated Annealing (SSA). Additionally, a hybrid algorithm combining Genetic Algorithms and SA, known as HGS, was proposed for solving the OPP problem. Li et al. (2013) made further improvements by strategically using specific PMU measurements and accessing critical system data to guarantee complete network observability. Abdulaziz et al. (2013) also employed Simulated Annealing to obtain optimal PMU placement for system observability at normal and abnormal conditions.

Differential Evolution

Differential Evolution (DE) operates through three primary mechanisms: mutation, crossover, and selection. This heuristic approach is highly effective for a broad range of objective functions, including those that are non-differentiable, nonlinear, or multi-modal. Notable advantages of DE include its simplicity of implementation, suitability for parallel computing, and strong convergence properties (Storn and Price, 1997).

Peng et al. (2010) proposed a multi-objective technique to the OPP problem using the nondominated Sorting Differential Evolution (NDSE) algorithm. This method integrates pareto-based non-dominated sorting with the standard DE framework to handle multiple conflicting objectives. The study applied NSDE to optimize PMU placement by maximizing measurement redundancy and accounting for potential voluntary PMU failures, while ensuring full observability of the power system. Rajasekhar et al. (2013) proposed a minimal PMU placement strategy based on Differential Evolution (DE) to evaluate network fault observability. Their study also examined the minimum number of PMUs required for full system observability by combining Integer Linear Programming (ILP) with DE, where DE efficiently guided the search toward optimal solutions.

Tabu Search

Tabu Search (TS) is a flexible metaheuristic algorithm that combines aspects of linear programming with heuristic strategies to tackle combinatorial optimization problems, particularly in areas such as scheduling and covering. Its defining feature is the tabu list, which records recently explored solutions and prevents the search from revisiting them. TS also employs mechanisms like the aspiration criterion, diversification strategies, and a wellstructured neighborhood definition for each solution state. When progress stalls or convergence is not achieved, a reset procedure is typically applied to restart the search process (Glover, 1989).

Peng et al. (2006) employed Tabu Search (TS) for the Optimal PMU Placement (OPP) problem, targeting complete system observability while maintaining sufficient measurement redundancy. Their approach was built on a linear state estimator model. To improve efficiency, Mesgarnejad (2008) introduced a Parallel Tabu Search variant, which reduced computation time by dividing the search domain into four parallel subspaces, each managed with its own tabu list.

Ant Colony Optimization

Ant Colony Optimization (ACO) is a bioinspired algorithm that simulates the foraging behavior of artificial ants to address complex optimization problems. These ants navigate the solution space by moving between neighboring states through a probabilistic, locally-informed decision process, which guides the search toward optimal solutions.

According to Li et al. (2013), ACO reduces computational complexity by efficiently identifying paths in graph-based structures. They further proposed an enhanced ACO variant for the Optimal PMU Placement (OPP) problem, aiming to achieve maximum monitoring of the power system with the fewest possible Phasor Measurement Units (PMUs), while also improving measurement redundancy. Arul et al. (2015) used a Fuzzified Binary Artificial Bee Colony algorithm to solve the multi-objective OPP problem minimizing PMU count and maximizing observability and voltage stability simultaneously.

Mutual Information

Mutual Information (MI), derived from information theory, has been applied to the Optimal PMU Placement (OPP) problem to achieve full network observability while accounting for uncertainties in power system states. This method evaluates the relationship between PMU measurement data and system states, as explained by (Mandava et al. 2015). In their study, a DC power flow model was adopted, since the MI metric could be analytically derived under this formulation. The analysis also considered multiple scenarios, such as the inclusion or exclusion of conventional measurements and the potential impacts of PMU failures.

Iterated Local Search

Iterated Local Search (ILS) focuses on exploring a restricted subspace consisting of locally optimal solutions, rather than traversing the entire solution space. Lourenço et al. (2001) introduced a hybrid heuristic approach in which a sequence of candidate solutions is generated, and the best one is selected through multiple randomized trials.

Hurtgen and Maun (2010) applied a two-stage framework to the OPP problem: first, identifying a feasible PMU configuration to guarantee system observability, and then employing ILS to minimize the number of PMUs while still ensuring effective monitoring of the power grid. Additionally, the PageRank Placement Algorithm (PPA) was integrated to evaluate the relative importance of each bus within the network. This method guaranteed full system observability even in the event of a PMU failure, thereby achieving a minimized yet resilient deployment strategy.

Immune Genetic Algorithm Similar to how vaccination and immune responses strengthen biological defenses against pathogens, incorporating local knowledge and prior information in the OPP problem parallels the "vaccination" concept in immune-based optimization algorithms. One such method, the Immune Genetic Algorithm (IGA), was developed to enhance solution optimality by employing two key operators: crossover and mutation.

Aminifar et al. (2009) applied IGA to the OPP problem by introducing three specially designed vaccines that guaranteed the topological observability of the power system. Their findings highlighted significant improvements in both convergence speed and computational efficiency. These advances were primarily attributed to a reproduction mechanism inspired by biological immune memory, which enabled the algorithm to evolve into a more effective and refined optimization tool.

Imperialistic Competition Algorithm

The Imperialistic Competition Algorithm (ICA) is a relatively recent metaheuristic optimization technique that has been successfully applied to a wide range of optimization problems. ICA begins with an initial population of candidate solutions, referred to as "countries," which are categorized as either imperialists or colonies. Its effectiveness and adaptability have been demonstrated across various optimization tasks using standard benchmark functions (AtashpazGargari and Lucas, 2007).

For the OPP problem, a binary adaptation of ICA known as the Binary Imperialistic Competition Algorithm (BICA) was introduced to address two key objectives: minimizing the number of PMUs required for complete observability and maximizing system redundancy to enhance measurement reliability. Moreover, BICA explicitly considers practical contingencies such as single-line outages, PMU failures, and the integration of zero-injection buses.

Biogeography Based Optimization

Biogeography-Based Optimization (BBO) is inspired by the natural migration patterns of species among habitats, where movements can lead to both species' proliferation and extinction. This algorithm addresses optimization problems by modeling dynamic fitness landscapes and incorporating species immigration and emigration processes. Simon (2008) introduced BBO with two fundamental operators- migration and mutation. Building on this framework, Wang et al. (2009) proposed a multi-objective extension, MOBBO, to tackle the OPP problem, aiming to minimize the number of PMUs required for full observability while simultaneously improving measurement redundancy to enhance state estimation accuracy. Since BBO does not always converge to a unique optimal solution, Jamuna and Swarup (2012) enhanced the approach by integrating non-dominated sorting and a crowding distance mechanism to extract Pareto-optimal solutions.

Pollination Algorithm

Hong-Shan et al. (2005) introduced a variant of the Flower Pollination Algorithm (FPA) to address the OPP problem. FPA has also been extended to handle multi-objective optimization, showing promising outcomes as noted by (Kekatos and Giannakis, 2012). The primary objective of this approach is to minimize the number of PMUs needed for complete observability of the power system, while simultaneously enhancing measurement redundancy across network buses.

Compared with other optimization techniques such as Binary Particle Swarm Optimization, the Greedy Algorithm, the Single Vertex Method, and Binary Integer Linear Programming, the FPA-based method demonstrated superior efficiency and effectiveness (Zhang et al., 2010).

Spanning Tree Search

Abdulaziz et al. (2013) used the Minimum Spanning Tree method to solve the OPP problem to ensure system observability. Prabhakar and Kumar (2014) proposed an efficient method that integrates the spanning tree technique with a tree search algorithm termed the Spanning Tree Search for addressing the OPP problem. This technique considers both single-channel and multichannel PMUs to get optimal observability of the power system. The study also analyzed the influence of observability depth on the total number of PMUs needed in the grid. The methodology involved constructing graph representations of the power network using an approach known as the spanning tree method, and the application of the tree search algorithm to identify optimal PMU placement locations. In addition, the method was extended to determine the fewest number of needed PMU channels for proper monitoring of the power grid. The proposed strategy was validated using IEEE standard bus systems within MATLAB, successfully achieving the dual objectives of minimizing PMU deployment and maximizing system observability.

Recursive Security Algorithm

Raj and Venkaiah (2015) introduced the Recursive Security Algorithm (RSA), which is based on the spanning tree search method and notable for its ability to generate multiple placement configurations. By evaluating different initial conditions, RSA identifies the most optimal PMU placement solutions.

Implemented through the "Power System Analysis Toolbox," the algorithm efficiently minimized the number of required PMUs while ensuring reliable monitoring of the power system.

Teaching–Learning-Based Optimization

Niknam et al. (2012) proposed the Teaching–Learning-Based Optimization (TLBO) method as an effective approach for addressing the OPP problem, both with and without zero-injection bus (ZIB) measurements. Unlike Genetic Algorithms (GA) and Binary Particle Swarm Optimization (BPSO), TLBO produces more consistent results and demonstrates higher efficiency (Niknam et al., 2012).

A major advantage of TLBO over GA and BPSO is its simplicity, as it requires fewer parameter settings, which improves ease of implementation and reduces the risk of performance degradation from complex constraints. Additionally, TLBO shows a faster convergence rate compared to GA and BPSO, as emphasized by (Zou et al. 2013). Expanding on this, Abdulaziz et al. (2013) applied TLBO to solve the multi-objective OPP problem, further confirming its effectiveness in handling complex optimization tasks.

IMPLEMENTATION DETAILS

The IEEE 14-bus, 30-bus, 57-bus ,118-bus, 300-bus, and the New England 39-bus test systems are most commonly employed as test cases in simulation studies. Among the systems with a larger scale, the Polish power system network comprising 2,746 buses and Entergy Corporation power network with 2,285 buses are also used (Nuqui & Phadke, 2005). In Nigeria, the 28-bus and 52-bus power system have also been used (Abdulkareem et al, 2021). A majority of the proposed methodologies for OPP problems have been developed and implemented using MATLAB.

Many of these MATLAB-based implementations utilize the MATLAB Optimization Toolbox. Additionally, several other PMU placement approaches are implemented using alternative optimization platforms such as CVX, CPLEX, and GAMS.

FUTURE RESEARCH

The following recommendations are proposed for future research: first, integrating dynamic grid conditions is essential to account for the time-varying nature of modern power systems. Moreover, addressing cyber-physical security concerns should be given top priority to ensure that PMU placement strategies also safeguard against cyber-attacks, vandalism, and communication failures. Another promising direction is the exploration of hybrid approaches, such as combining semidefinite programming (SDP) with metaheuristic techniques or machine learning, which could enhance scalability, adaptability, and overall efficiency. Finally, greater attention should be devoted to emerging developments in power grid operations including generation, transmission, distribution, and utilization while also considering geographical and communication constraints, all of which present valuable opportunities for future investigation.

CONCLUSION

One of the key challenges in modern power system monitoring lies in transitioning from conventional power networks to smart grids while maintaining full observability or at least achieving the highest possible level of observability through the strategic deployment of PMUs. At the core of the PMU placement problem is determining the minimum number of units required to guarantee sufficient system observability.

This study provides a comprehensive overview of PMU placement strategies within both single- and multi-objective optimization framework. It reviews a wide range of methodologies designed to deliver structured and practical solutions to this critical challenge in power system monitoring. These approaches aim to balance cost, observability, and system reliability, offering valuable insights for power system operators. In addition, the work examines the fundamental techniques applied in Optimal PMU Placement (OPP) and presents a broad perspective on the problem. It establishes a solid foundation for future research by identifying key gaps in the literature and encouraging the development of effective, scalable, and practical solutions to meet the evolving requirements of nextgeneration power systems.

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