ANALYSIS OF A PIPE RESTING ON WINKLER FOUNDATION AND CONVEYING A NON-NEWTONIAN FLUID

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ABSTRACT

Fluid conveyance pipes are subject to vibration and the attendant dynamic stresses lead to instability and sometimes failure. The fluid inside the pipe dynamically interacts with the motion of the pipe, therefore, the material properties of the fluid are expected to play important roles on it. This paper analyzed the deformation of a pipe resting on Winkler foundation and conveying a non-Newtonian fluid. Euler-Bernoullis beam and plug flow models are employed in this work. The resulting transverse differential equation was non-dimensionalized, discretized and solved by means of the finite element method. Effects of the fluid materials and other flow conditions were computed and their implications on the pipe integrity analyzed. Natural frequency increased with flow index, Winkler foundation and mass ratios. Critical velocity also increased with Winkler foundation.

Keywords: fluid, pipe, deformation, Winkler foundation, instability

INTRODUCTION

Pipes are widely used in many industrial fields. The fluid flow and pipes are interactive systems, and their interaction is dynamic. These systems are coupled by the force exerted on the pipe by the fluid (Abid Al-Sahib *et al*., 2010). Flow-induced vibration analysis of pipes conveying fluid has been one of the attractive subjects in structural dynamics (Liu and Xuan, 2010). The free vibrations of pipes conveying fluid was studied by Huang (1974) and reported in Abid Al-Sahib *et al.* (2010*)*, taking into account, the effects of rotary inertia on both the fluids and the pipes, the shear deformation of the pipes and the lateral inertia force due to the moving fluids. Naguleswaran (2002a, 2002b and 2004) obtained an approximate solution to the transverse vibration of the uniform Euler-Bernoulli beam under linearly varying axial force. The author also extended this approach to find the natural frequencies, sensitivity and mode shapes of Euler- Bernoulli beam with up to three step changes in cross-section. Dian *et al.* (1998) analyzed the free lateral vibration of thin annular with variable thickness and circular plates. The study adopted the finite element method to obtain the natural frequencies and mode shapes of the axisymmetric and non axisymmetric thin annular. The results showed that the finite element method was an efficient and convenient tool for analyzing the lateral vibration of annular and circular composite plates with variable thickness. The effect of induced vibration of a simply supported pipe conveying fluid with a restriction was investigated theoretically and experimentally by *Alaa* (2001), transfer matrix approach was implemented to describe the dynamic response of a pipe conveying fluid and a numerical technique for solving two dimensional incompressible steady viscous flow for the range of Reynolds number($5<Re<1000$). He concluded that the fluid flow through a pipe with restriction affected the dynamic behavior of the pipe in addition to the flow field structure due to induced vibration. Olunloyo *et al.* (2007) studied the dynamics of offshore fluid conveying pipe and pipe walking phenomenon alongside the effect of elevated temperature and concluded that the role of the transient solution may not be as central for pipe walking as was hitherto believed as there are other significant contributions emanating from some of the other parameters.

Shen *et al*. (2009) based on Timoshenko beam model studied the band gap properties of the flexural vibration for periodic pipe system conveying fluid using the transfer matrix method. These methods have proved to be effective in analyzing flow-induced vibration of certain pipes. It is well known that analysis of pipe dynamics could be conducted based on the energy-based approach according to Hamiltonian principle (Stangl et al., 2005; Stangl and Irschik, 2006). Xu *et al*. (2010) proposed the analytical expression of natural frequencies of fluid-conveying pipes with the help of homotopy perturbation method. Those calculated frequencies were in good agreement with experimental results.

Reddy and Wang (2004) derived equations of motion governing the deformation of fluid-conveying beams using the kinematic assumptions of both Euler-Bernoulli and Timoshenko beam theories. The formulation accounted for geometric nonlinearity in the Von Karman sense and contributions of fluid velocity to the kinetic energy as well as to the body forces. Finite element models of the resulting non-linear equations of motion were also presented. Kuiper and Metrikine (2004) proved analytically the stability of a clamped-pinned pipe conveying fluid at a low speed. A tensioned Euler-Bernoulli beam in combination with a plug flow was used as a model. The stability was studied employing a D-decomposition method.

The three-dimensional nonlinear differential equations of a fluid-conveying pipe undergoing overall motions were derived by Meng *et al.* (2011) based on Kane's equation and the Ritz method taking into consideration the effect of the internal and external fluids. They obtained the time histories for the displacements using the incremental harmonic balance method.

The Finite Element Method (FEM) according to Gunakala *et al.* (2004) is one of the most powerful tools used in structural analysis. Finite Element Analysis is based on the premise that an approximate solution to any complex engineering problem can be reached by subdividing a larger complex structure into smaller non-overlapping components of simple geometry called finite elements or elements .Complex partial differential equations that describe these structures can be reduced to a set of linear equations that can easily be solved using this method.

Abraham (2001) studied the vibration and stability of straight pipe systems conveying fluid, either steady or fluctuating flow. He considered supports of different type and positions and concluded that the support position and values had a significance on dynamic characteristics of the pipe. For pipes of a finite length, the dynamical behaviour depends strongly on the type of boundary conditions at both ends (Lee and Mote, 1997; Kuiper and Metrikine, 2004). They agreed that distinction should be made between the type of supports (fixed, simple, free, inertial, etc.) and their location (upstream or downstream). It is known that a tensioned pipe conveying fluid and having symmetrical boundary conditions (*clamped-clamped, pinned-pinned*, etc.) behaves as a conservative gyroscopic system (Païdoussis, 1998; Kuiper and Metrikine, 2004), implying that the total energy of the system varies periodically with time. Païdoussis (2004) studied numerically pinned-clamped and clamped-pinned pipes conveying fluid. He found that to predict the dynamical behaviour of the clamped-pinned pipe, even 8 significant-figure accuracy was not good enough.

In practice, long, cross-country pipelines rest on an elastic medium such as a soil, and hence, a model of the soil medium must be included in the analysis. The Winkler model, in which soil is represented by a series of constant stiffness, closely spaced linear springs, is a very popular model of the soil employed in many studies, perhaps because it is a simple linear model. Many researchers, Stein & Tobriner, (1970); Lottati and Kornecki, (1986); Dermendjian-Ivanova, (1992) and Chary *et al*., (1993) studied fluid-conveying pipes resting on elastic foundation. Becker *et al*. (1978) as reported in Ibrahim (2011) found that the critical flow velocity of a pipe on a Winkler foundation is higher than the critical flow velocity of the same pipe without a foundation. Doaré *et al*. (2002) also studied instability of fluid conveying pipes on Winkler type foundation.

High molecular weight liquids are usually non-Newtonian (Subramanian & Shankar, 2003). When the viscosity decreases with increasing shear rate, the fluid is shear-thinning. However, many shear-thinning fluids exhibit Newtonian behaviour at either extremely low or high shear rates (Herh *et al.,* 2003). Thus, the Navier-Stokes equations of incompressible viscous non-Newtonian fluid, Maxwell's equations of electrodynamics and the energy equation are the basic equations of motion for incompressible, viscous conducting non-Newtonian power-law fluids (Subramanian & Shankar, 2003).

GOVERNING DIFFERENTIAL EQUATION

Figure 1 shows the schematic diagram of the system with its axis corresponding with the longitudinal direction. The pipe idealized as an elastic beam is clamped at both ends and having dimensions given by the length L, the inner radius r. The pipe is assumed to be sufficiently slender that it can be modeled as the Euler-Bernoulli beam. The fluid being conveyed is assumed to be an incompressible non-Newtonian fluid and plug flow is also assumed.

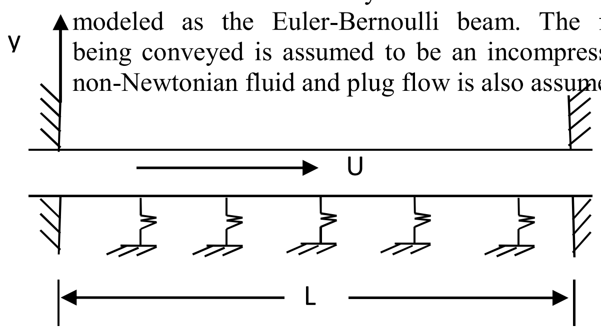

Figure1. Schematic diagram of a straight fluid-conveying pipe with both ends fixed

According to Bird *et al.* (2007) on Power law, shear stress on a fluid flowing in a pipe is given by

$$\tau_{rx} = m\left(\frac{-dv_x}{dr}\right)^{\nu} \tag{1}$$

where,

m = non-Newtonian viscosity

v_x = velocity of flow in x-direction

r = internal radius of the pipe

υ = fluid flow index

But for a horizontal flow under the influence of pressure gradient,

$$\tau_{rx} = -\left(\frac{P_L - P_o}{2L}\right)r \tag{2}$$

where,

P_o = pressure at entry

P_L = pressure at exit

L = length of pipe

Substituting Eq.(2) into Eq.(1) gives

$$-\left(\frac{P_L - P_o}{2L}\right)r = m\left(\frac{-dv_x}{dr}\right)^{\nu}$$

or

$$\left(\frac{P_L - P_o}{2L}\right)^{\frac{1}{\nu}}\left(\frac{r}{m}\right)^{\frac{1}{\nu}} = \frac{dv_x}{dr} \tag{3}$$

Integrating Eq. (3) gives

$$v_x = \left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}}\frac{\nu}{1+\nu}r^{\frac{1+\nu}{\nu}} + C \tag{4}$$

at r = R, $v_x = 0$,

$$C = -\left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}}\frac{\nu}{1+\nu}R^{\frac{1+\nu}{\nu}} \tag{5}$$

Substituting Eq.(5) into Eq.(4) gives

$$U = v_x = \left(\left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}}\frac{\nu}{1+\nu}r^{\frac{\nu+1}{\nu}}\right) - \left(\left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}}\frac{\nu}{1+\nu}R^{\frac{\nu+1}{\nu}}\right) \tag{6}$$

$$= \left(\left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}}\frac{\nu}{1+\nu}\right)(r-R)^{\frac{\nu+1}{\nu}}$$

$$U = -\left(\left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}}\frac{\nu}{1+\nu}\right)R^{\frac{\nu+1}{\nu}}\left(1-\frac{r}{R}\right)^{\frac{\nu+1}{\nu}} \tag{7}$$

The maximum velocity v_{max}, is the velocity at the centre of the pipe (r = 0)

$$v_{\max} = -\left(\left(\frac{P_L - P_o}{2Lm}\right)^{\frac{1}{\nu}} \frac{\nu}{1+\nu}\right) R^{\frac{\nu+1}{\nu}} \tag{8}$$

Eq.(8) makes it possible for Eq.(6) to be written as

$$U = v_{\max}\left(1 - \left(\frac{r}{R}\right)^{\frac{\nu+1}{\nu}}\right) \tag{9}$$

Substituting Eq. (9) in Euler-Bernoulli's Equation for fluid-pipe structure gives

$$EI\frac{\partial^4 w}{\partial x^4} + M\frac{\partial^2 w}{\partial t^2} + 2m_f v_{\max}\left(1 - \left(\frac{r}{R}\right)^{\frac{\nu+1}{\nu}}\right)\frac{\partial^2 w}{\partial t \partial x} + m_f v_{\max}\left(1 - \left(\frac{r}{R}\right)^{\frac{\nu+1}{\nu}}\right)^2 \frac{\partial^2 w}{\partial x^2} = 0 \tag{10}$$

when the structure is lying on Winkler foundation, Eq. (10) becomes

$$EI\frac{\partial^4 w}{\partial x^4} + M\frac{\partial^2 w}{\partial t^2} + 2m_f v_{\max}\left(1 - \left(\frac{r}{R}\right)^{\frac{\nu+1}{\nu}}\right)\frac{\partial^2 w}{\partial t \partial x} + m_f v_{\max}\left(1 - \left(\frac{r}{R}\right)^{\frac{\nu+1}{\nu}}\right)^2 \frac{\partial^2 w}{\partial x^2} + Kw = 0 \tag{11}$$

where,
EI= flexural stiffness,
$M = m_f + m_p$ = mass of the pipe and the fluid flowing in it
K = soil stiffness

The associated boundary conditions are given by

$$w(0,t) = \frac{\partial w(0,t)}{\partial x} = 0 \text{ and } w(L,t) = \frac{\partial w(L,t)}{\partial x} = 0 \tag{12}$$

Eq. (11) is non-dimensionalized to give

$$\frac{\partial^4 \overline{w}}{\partial \overline{x}^4} + \frac{\partial^2 \overline{w}}{\partial \overline{t}^2} + 2\delta \overline{v}_{\max}\left(1 - \left(\frac{\overline{r}}{\overline{R}}\right)^{\frac{\nu+1}{\nu}}\right)\frac{\partial^2 \overline{w}}{\partial \overline{t} \partial \overline{x}} + \delta \overline{v}_{\max}{}^2\left(1 - \left(\frac{\overline{r}}{\overline{R}}\right)^{\frac{\nu+1}{\nu}}\right)^2 \frac{\partial^2 \overline{w}}{\partial \overline{x}^2} + \overline{K}\overline{w} = 0 \tag{13}$$

where, $\overline{x} = \frac{x}{L}, \overline{w} = \frac{w}{L}, \overline{r} = \frac{r}{L}, \overline{R} = \frac{R}{L}, \delta = \frac{m_f}{M} \overline{v}_{\max} = v_{\max} L\sqrt{\frac{M}{EI}}, \overline{t} = \frac{t}{L^2}\sqrt{\frac{EI}{M}}$

3.FINITE ELEMENT ANALYSIS (DISCRETIZATION OF THE GOVERNING DIFFERENTIAL EQUATION)

The first step in the finite element analysis is the discretization of the domain. Here, the domain of the problem is divided into a finite set of line elements and each element having at least two end nodes.

$$\overline{w}(\overline{x},\overline{t}) = \sum_{j=0}^{n} u^e{}_j(\overline{t})\psi^e{}_j(\overline{x}) = \sum_{j=0}^{n} \left(u^s{}_j\right)^e \psi^e{}_j(\overline{x}) \quad (s = 1,2,.....) \tag{14}$$

where,
$\left(u^s{}_j\right)^e$ is the value of $u(\overline{x},\overline{t})$ at time $t = t_s$ and node j of the element Ω_e.
Substituting $w = \psi_i(\overline{x})$ into Eq. (13) leads to

$$\int_0^1 \left(\frac{d^2\psi_i}{d\bar{x}^2}\right)\left(\sum_{j=1}^{n} u_j \frac{d^2\psi_j}{d\bar{x}^2}\right) - \bar{v}_{\max}{}^2 \left(1-\left(\frac{\bar{r}}{\bar{R}}\right)^{\frac{\nu+1}{\nu}}\right)^2 \int_0^1 \frac{d\psi_i}{d\bar{x}}\left(\sum_{j=1}^{n} u_j \frac{d\psi_j}{d\bar{x}}\right)$$

$$+ 2\delta\bar{v}_{\max}\left(1-\left(\frac{\bar{r}}{\bar{R}}\right)^{\frac{\nu+1}{\nu}}\right)\int_0^1 \frac{d\psi_i}{d\bar{x}}\left(\sum_{j=1}^{n}\frac{d\psi_j}{d\bar{t}}\psi_j\right) + \int_0^1 \psi_i\left(\sum_{j=1}^{n}\frac{d^2u_j}{d\bar{t}^2}\psi_j\right) + \bar{K}\int_0^1 \psi_i\left(\sum_{j=1}^{n} u_j\psi_j\right) - \psi_i f \quad (15)$$

$$-Q_1\psi_i(0) - Q_3\psi_i(1) - Q_2\left(-\frac{d\psi_i}{d\bar{x}}\right)\Bigg|_0 - Q_4\left(-\frac{d\psi_i}{d\bar{x}}\right)\Bigg|_1 = 0$$

$$\sum_{j=1}^{n}\left(K^1{}_{ij} - K^2{}_{ij} + K^3{}_{ij}\right)u_j + C_{ij}\frac{du_j}{d\bar{t}} + M_{ij}\frac{d^2u_j}{d\bar{t}^2} - F_i \quad (16)$$

where M is the mass matrix, C is the matrix relating to the gyroscopic force, K^1 is the structural stiffness matrix, K^2 is the matrix related to centrifugal force in the fluid and K^3 is the matrix related to the Winkler foundation.

In vector-matrix form, Eq. (16) becomes

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \quad (17)$$

4. ANALYSIS OF THE NATURAL FREQUENCIES

When the pipe is in steady state, Eq. (17) can be written as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = 0 \quad (18)$$

Let $u(\bar{x},\bar{t}) = U(\bar{x})T(\bar{t})$ (19)

Substituting Eq.(19) into Eq.(18) gives

$$[M]\ddot{T} + [C]\dot{T} + [K]T = 0 \quad (20)$$

The natural frequencies of the system can be obtained from Eq. (20) by transforming it into

$$A\dot{Y}(\bar{t}) + BY(\bar{t}) = 0 \quad (19)$$

where $A = \begin{bmatrix} O & I \\ M & O \end{bmatrix}$, $B = \begin{bmatrix} -I & O \\ C & K \end{bmatrix}$, $Y(\bar{t}) = \begin{Bmatrix} \dot{u}(\bar{t}) \\ u(\bar{t}) \end{Bmatrix}$ (20)

and I, O are identity and zero matrices respectively

If the solution of Eq. (19) is assumed to be

$Y(\bar{t}) = Y_o e^{\lambda_n \bar{t}}$, the complex eigenvalue λ_n can be computed from

$$\det(B + \lambda_n A) = 0 \quad (21)$$

And the natural frequencies can now be obtained from the eigenvalues

RESULTS

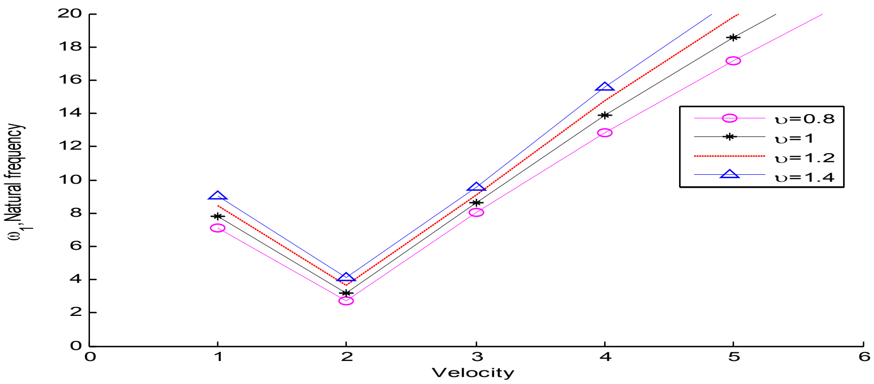

Figure 2: Natural frequency ω_1 profile as a function of flow index for the case $L = 5m, \gamma = 0, \delta = 0.4$

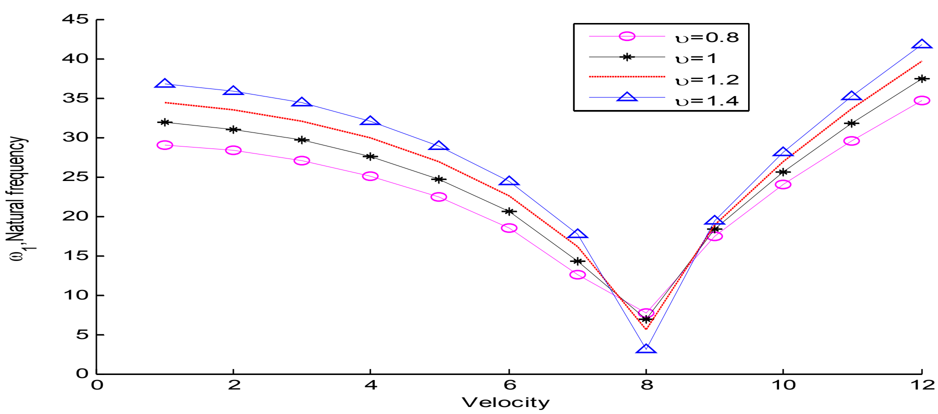

Figure 3: Natural frequency ω_1 profile as a function of flow index for the case $L = 5m, \gamma = 2, \delta = 0.4$

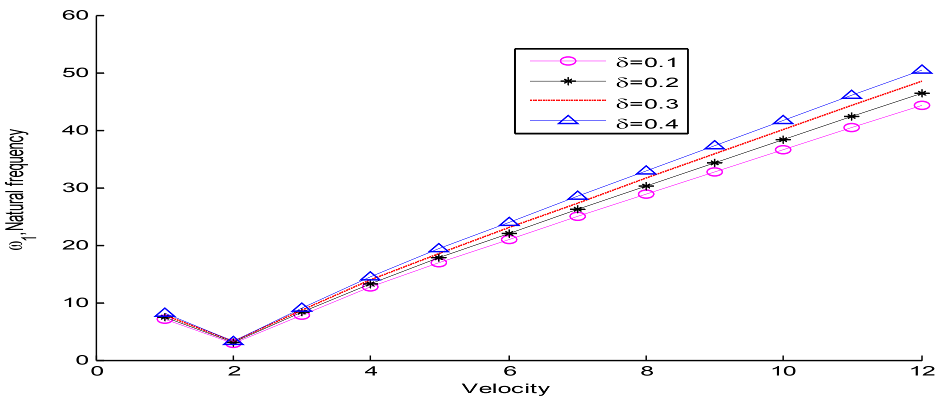

Figure 4: Natural frequency ω_1 profile as a function of mass ratio for the case $L = 5m, \gamma = 0, \upsilon = 1$

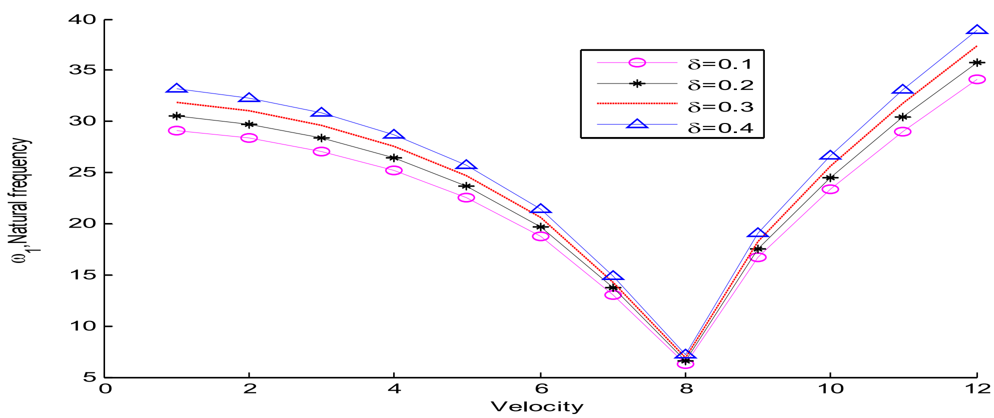

Figure 5: Natural frequency ω_1 profile as a function of mass ratio for the case $L = 5m, \gamma = 2, \upsilon = 1$

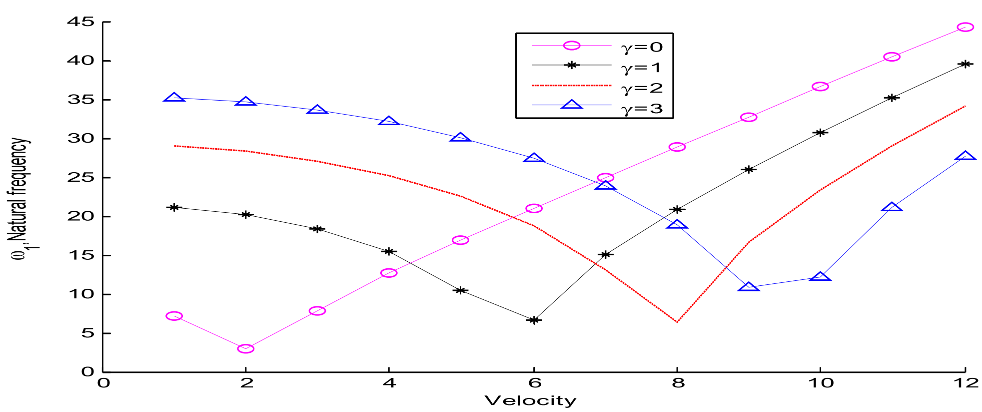

Figure 6: Natural frequency ω_1 profile as a function of foundation stiffness for the case $L = 5m, \delta = 0.4, \upsilon = 1$

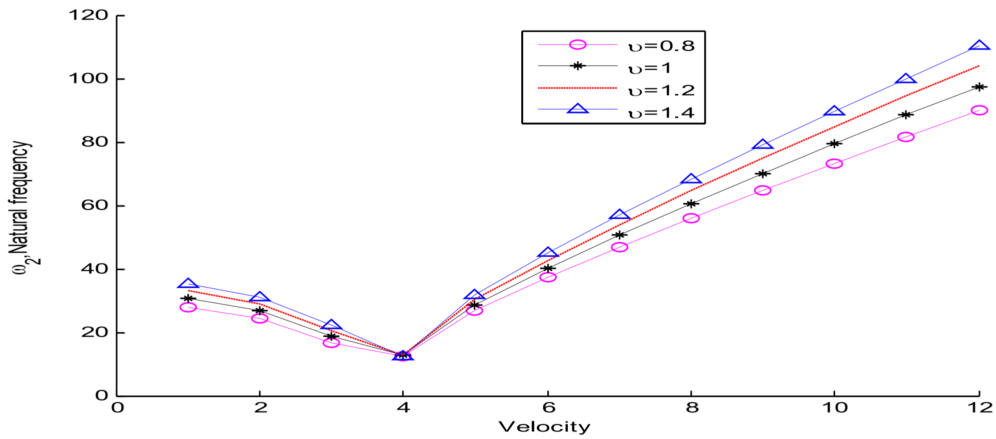

Figure 7: Natural frequency ω_2 profile as a function of flow index for the case
$L = 5m, \gamma = 0, \delta = 0.4$

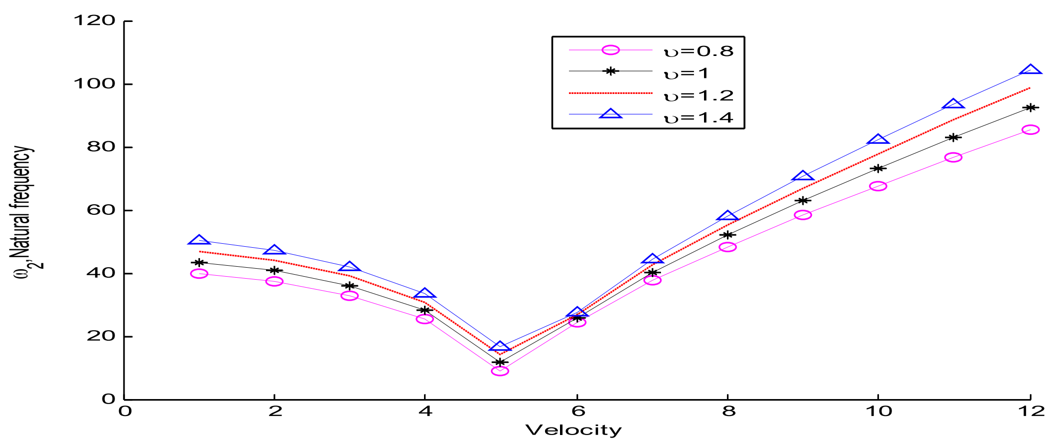

Figure 8: Natural frequency ω_1 profile as a function of flow index for the case
$L = 5m, \gamma = 2, \delta = 0.4$

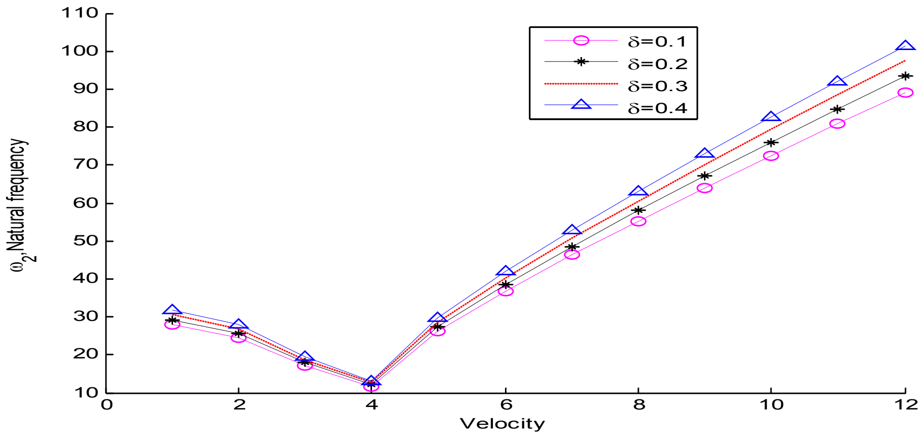

Figure 9: Natural frequency ω_2 profile as a function of mass ratio for the case $L = 5m, \gamma = 0, \upsilon = 1$

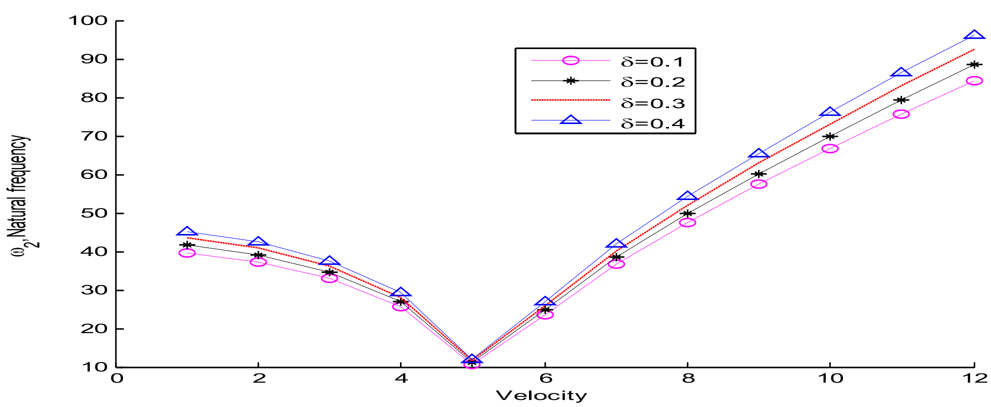

Figure 10: Natural frequency ω_2 profile as a function of mass ratio for the case $L = 5m, \gamma = 2, \upsilon = 1$

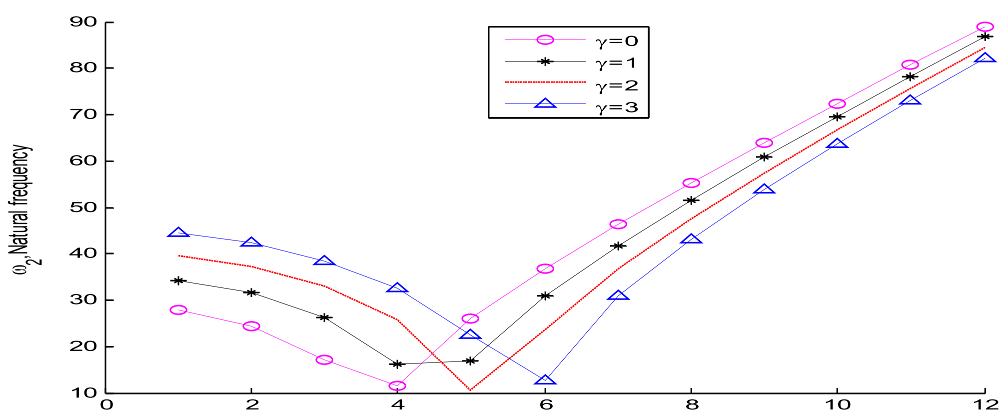

Figure 11: Natural frequency ω_2 profile as a function of foundation stiffness for the case

$$L = 5m, \delta = 0.4, \upsilon = 1$$

DISCUSSION OF RESULTS

This paper analyzed a pipe resting on Winkler foundation and conveying a non-Newtonian fluid. The pipe was idealized as an elastic beam clamped on both ends and resting on Winkler foundation through which a non-Newtonian fluid was flowing. A model was obtained for the fluid-pipe-foundation system.

Simulations of a particular pipeline design was carried out based on characteristic values of some of the fluid and mechanical parameters that govern the interaction. The following physical parameters were considered in this work:

$$\rho_p = 7800kg/m^3, \rho_f = 1000kg/m^3, m =, R_o = 26mm, R_i = 24mm, E = 210GPa, \gamma = 0,1,2,3$$

In this study, the first two natural frequencies were considered and generally, the two natural frequencies decrease with velocity of conveyance. This agrees with Ozhan and Pakdermirli (2013), Kuye (2013) and Doaré *et al*. (2002). The bigger the natural frequency of a system the better it will be able to withstand impressed vibration without failure.

Figure 2 - 11 display the behavior of the first natural frequency ω_1 with velocity for different flow parameters. In Figure 2, natural frequency ω_1 increases with flow index υ for the case of $\gamma = 0$. This indicates that if the flow index range is pseudo-plastic (pp)<Newtonian (N)<dilatant (d) and the index for Newtonian is taken to be 1, then $\omega_{1(pp)} \langle \omega_{1(N)} (N) \langle \omega_{1(d)}$. The critical velocity, v_{crt}, is the velocity at which the natural frequency drops to its lowest. In this situation, v_{crt} occurs at v = 2m/s. The highest ω_1 recorded is 10.

Figure 3 displays a similar pattern with that of Figure 1 for varying mass ratio, the behavior of the natural frequency ω_1 increases with mass ratio υ for $\gamma = 0$.

Figures 4 and 5 have similar patterns with Figures 1 and 2. Natural frequency ω_1 increases with both the flow index υ and the mass ratio δ for $\gamma = 2$.

In Figure 6, natural frequency ω_1 and v_{crt} increase with γ. This agrees with Doaré *et al*. (2002), Ozhan and Pakdemirli, (2013), Becker *et al*. (1978) and Ibrahim (2011). Figures 7 – 11 replicate similar patterns with Figures 2 – 6 for both $\gamma = 0$ and $\gamma = 2$ for the second natural frequencies ω_2 but with higher values compared with ω_1.

CONCLUSION

This paper analyzed a pipe resting on Winkler foundation and conveying a non-Newtonian fluid. It can be concluded that natural frequency increased with flow index, Winkler foundation and mass ratios. Critical velocity also increased with Winkler foundation. Any pipe conveying a fluid should be made to rest on a foundation for good performance. The same velocity must not be used to convey all fluids, the flow indices must be considered to safeguard resonance that can lead to failure of the pipe.

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