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OPTIMAL ALLOCATION OF PIPE DIAMETERS IN PIPE NETWORKS USING MODIFIED ASSIGNMENT PROBLEM APPROACH

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ABSTRACT

Water supply to areas of need via pipe network has been a problem since the pipes layout involved may be complex and thereby making the determination of pipe diameters for cost minimization difficult. In this paper, attempt was made on the determination of optimal diameters of pipe that minimizes pipe network cost. The work involves the use of a linear programming optimization technique formulated in a modified assignment problem approach. This was applied to a problem previously worked upon with some other techniques to ascertain the strength of this present approach implemented on MATLAB 6.5 version computing environment.

The minimum cost unit of 41900 for the eight-pipe, one reservoir and two loop distribution system obtained was the same with that of the "best run" in the previous reported works. Also, 12 functions evaluation in 53 seconds on Pentium 233 MHz processor computer produced this optimal condition as against the 1372 evaluations in 7 minutes on Pentium 100MHz processor computer for the previous "best run". These indicate the efficiency and the effectiveness of the studied approach.

Keywords: Pipe Networks, Head loss, Nodal head, Efficiency, Optimal Pipe Diameter.

INTRODUCTION

Transportation of liquids via networks of pipe has become an integral part of both upstream and downstream petroleum sectors; management of which can either boost or reduce the sectors earnings. Apart from this, municipal water distribution systems represent a major portion of the investment in urban infrastructure and a critical component of public works. This normally have the goal of designing water distribution systems to deliver potable water over spatially extensive areas in required quantities and under satisfactory pressures. In addition to these goals, cost-effectiveness and reliability in system design are also important.

The designs of this type of water distribution systems are inherently complex because they are large-scale and spatially extensive. It also composed of multiple pipe loops to maintain satisfactory levels of redundancy for system reliability and it is governed by nonlinear hydraulic equations. Their designs mostly include complex hydraulic devices such as valves and pumps and are impacted by pumping and energy requirements. Other characteristics of this system includes, the complication by numerous layout, pipe sizing, and pumping alternatives, the influence of tradeoff between capital investment, operations and maintenance costs during the design process (Nicklow,2000).

The optimal design of municipal water distribution systems is a challenging optimization problem for the following reasons, firstly, the system optimization requires an imbedded hydraulic simulation model for pressurized, looped pipe networks and the decision variables are discrete, since pipe sizes must be selected from commercially available sets. Secondly, the combinatorial problems involving discrete variables are considered NP-hard in optimization theory and the optimization problem can be highly nonlinear due to nonlinear hydraulic models and pump characteristic curves. Also, the optimization problem are regarded as stochastic due to uncertain demand loadings and system reliability issues and finally, pressure constraints must be directly included in the optimization (Nicklow,2000).

Previous researches report the formulation of this type of problem on a component basis in a non linear manner whose solutions are been considered NP-hard in optimization theory (Wood and Rayes, 1981). But, this paper puts forward a linear programming approach using modified assignment problem optimization techniques with the determination of the optimal diameters of pipes in a network with a predetermined layout. This modification relaxes an assignment problem condition of summation of decision variables along the row being equal to one. This relaxed condition now gives room for summation greater than unity to allow a particular pipe diameter being chosen more than once if it will really reduce the cost. This also includes providing the pressure and quantity of water required at every demand node. A case study previously solved with non linear approaches obtained from literature is used to test the efficacy of the proposed approach.

Previous Design and Optimization of Pipe Networks

Optimization of pipe networks has gained much attention in the past few decades. Numerous algorithms are being tested on distribution systems by researchers to get the most reliable solutions, using the least computational time possible. Linear programming (LP), nonlinear programming (NLP), mixed-integer linear programming (MILP), mixed-integer nonlinear programming (MINLP), as well as fuzzy logic (FL), stochastic dynamic programming (SDP) and the genetic algorithm (GA) are the most promising methods.

Creasey (1988) reviewed the appropriate mathematical techniques to solve the problem of operational optimisation for water distribution networks, insisting particularly on the pump-scheduling problem. He also stated that integer-based programming seems to be the only way to achieve savings for a wide range of network sizes and complexities because of their ability to handle high non linearity resulting within short time. Examples of application of dynamic programming on small-scale systems can be found in Rao and Bree (1977), Wood and Rayes (1981), Goldberg and Kuo (1987), and Coulbeck (1988).Also, this handles non linearity but becomes impractical for large size networks due to extraordinarily wide search space and consequently the enormous computational time.

With the availability of linear programming (LP) algorithms that were more robust and efficient, several papers on the subject were published. Jowitt and Germanopoulos (1992) produced the most significant applications in the field of LP techniques applied to the pump-scheduling problem. One of the latest approaches consists of taking into account the non-linear relationships, which are part of any pump-scheduling problem. Kessler and Shamir (1989) used the linear programming gradient (LPG) method as an extension of the method proposed by Alperovits and Shamir (1977). Also a two-phase decomposition method was used extending that of Alperovits and Shamir (1977) to non-linear modelling. Though split pipe solutions obtained in the above cases are cheaper, some of the results obtained were not practical and some others were not feasible. In addition, some of these methods impose a restriction on the type of the hydraulic component of the network which does not give room for global optimality.

The appearance of improved non-linear programming (NLP) algorithms, convinced researchers to rather apply NLP techniques to solve the operational optimisation problem. Chase and Ormsbee 1989), Lansey and Zhong (1990) and Brion and Mays (1991) linked network-simulation models with non-linear optimization algorithms to determine optimal operations. One commonly used approach in operational optimization problems has been the utilization of a MINLP algorithm (Pahor and Kravanja, 1995; Grossman, 1996; Bruno et al., 1998; Zamora and Grossman, 1998). Though most of these approaches are well suited for non linear conditions but becomes problem when storage space and computational time are considered.

Also, genetic algorithms have been applied in the problem of pipe network optimization (Savic and W alters ,1997). He applied both simple genetic algorithm (SGA) and improved GA, with various enhancements based on the nature of the problem, and reported promising solutions for problems from literature. This approach is charaterised by uncertainty about the termination of the search and the absence of guarantee for the global optimum.

Problem Formulation

Pipe network, especially in water distribution, is system containing pipes, reservoirs, pumps, valves of different types, which are connected to each other to provide water to consumers at different nodes. Generally, the hydraulic requirements on any network of pipes impose two constraints: the continuity constraint; for n nodes in the network, this constraint can be written as:

$$\sum_{i=1}^{n} Q_i = 0 \qquad (1)$$

where Q_i represents the discharges into or out of the node i (sign included).

The second hydraulic constraint is the energy constraint according to which the total head loss around any loop must add up to zero or is equal to the energy delivered by a pump if there is any:

$$\sum h_f = 0 \qquad (2)$$

where h_f is the head loss due to friction in a pipe. This embeds the fact that the head loss in any pipe, which is a function of its diameter, length and hydraulic properties, must be equal to the difference in the nodal heads. This constraint makes the problem highly non-linear owing to the nature of the equation that relates frictional head loss and flow. The equation can generally be written as

$$h_f = \frac{aQ^b}{D^c} \qquad (3)$$

where a is coefficient depending on length, roughness, etc, b is discharge exponent and c is exponent of pipe diameter (D) which is very close to 5 in most head loss equations.

Considering the diameters of the pipes in the network as decision variables, Abe and Solomatine (1998) reported that the problem can be considered as a parameter optimization problem with dimension equal to the number of pipes in the network. Market constraints, however, dictate the use of commercially available (discrete) pipe diameters. With this constraint, the problem can be formulated as a combinatorial optimization problem. The minimum head requirement at the demand nodes is taken as a constraint for the choice of pipe diameters.

Formulating the above constraints in such a manner suited for assignment problem techniques calls for sets of linear equations. For a problem to fit the definition of an assignment problem, such kinds of applications need to be formulated in ways that satisfy the following assumptions:

- The number of assignees and the number of tasks are the same
- Each assignee is to be assigned to exactly one task.
- Each task is to be performed by exactly one assignee.
- There is a cost c_{ij} associated with assignee i performing task j.
- The objective is to determine how all n assignment should be made in order to minimize the total cost (Hillier and Lieberman,1995).

The mathematical model for assignment problem uses the following decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if assignee i performs task j} \\ 0 & \text{if not} \end{cases} \quad (4)$$

But for pipe network problems where the number of commercially available pipes diameters i are not usually the same with the number of pipes j needed in a network resulting in an n x m matrices of assignment variables, it means that the number of assignees and the number of tasks are not the same. This problem can be solved to make it suitable for assignment problem approach by repeating the ith pipe diameter (assignment) in m-times for jth task to make-up but still carrying its assigned parameters. The jth task is also repeated in n-times on ith assignment level for possible multiple choice of an assignee to tasks of that ith assignment. This gives room for multiple choice of a particular pipe diameter in the network. The resulting matrix here is a nm x nm type that is suited for this solution approach.

Objective function

The objective function to be minimized by the optimization approach is the cost of the network which is calculated based on the cost per unit length associated with the diameter and the length of the pipe. But in a situation where the number of assignee is more than the task or vise versa, dummy variables should be introduced to make up for the deficiency so that a square matrix can be obtained (i.e. n = m). It should also be noted that unit cost higher than the highest on the original cost distribution should be assigned to the dummy variables in order to eliminate them from emerging as the optimal variables. With all these ,the objective function can now be written as:

$$\text{Minimize } C_a = \sum_{i=1}^{n} \sum_{j=1}^{m} c(D_i) x_{ij} L_i \quad (5)$$

Where n is the number of commercially available pipe diameters, m is the number of pipes in the network and $c(D_i)$ is the cost per unit length of the jth pipe with diameter D_i and length L_i.

Constraints

The above objective function is subject to the two hydraulic constraints in equations 1 and 2 which are also linearised by the introduction of assignment variables, also included is the minimum nodal head constraint. To ensure its adherence to the assignment problem format when solving the resulting linear programming problem, the following constraints are also introduced:

$$\text{For i = 1 to n, then} \quad \sum_{j=1}^{m} x_{ij} = 1 \quad (6)$$

$$\text{For j = 1 to m, then} \quad \sum_{i=1}^{n} x_{ij} \le n \quad (7)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} = n \quad (8)$$

$$x_{ij} \ge 0 \quad \text{(for all i and j)} \quad (9)$$

(x_{ij} binary, for all i and j)

Once the above equations are properly formed and the required parameter solved for in any algebraic solver, the problem can then be solved using any linear programming package such as MATLAB 6.5 version computing environment to obtain the required result.

Selected Case Study

The selected case study is the one reported by Abe and Solomatine (1998) for a two loop network with 8 pipes,7 nodes and one reservoir as shown in Figure 1.This was chosen in order to test the effectiveness and efficiency of the solution approach being examined since data on the previous attempts are available therein. All the pipes are 1000m long and Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30 m. There are 14 commercially available pipe diameters and their associated cost as shown on Table 1. Table 2 shows the node data for the two loop network.

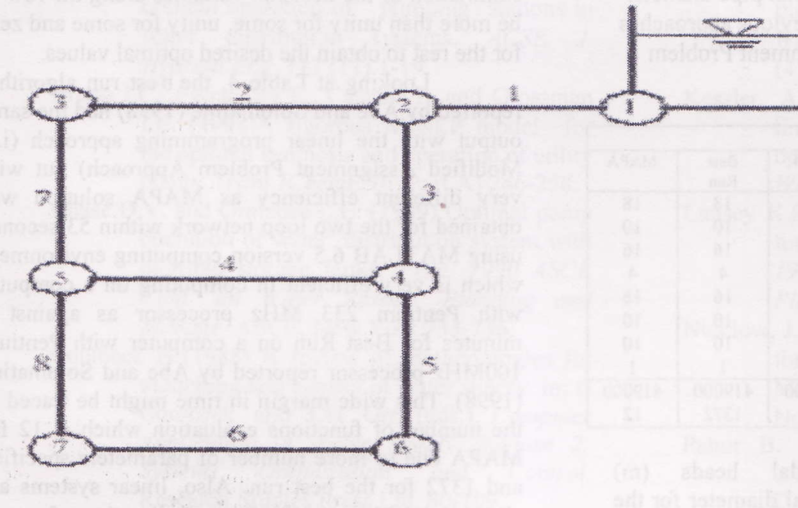

Figure 1: The two-loop network (*Source: Abe and Solomatine ,1998*)

Table 1: Cost data for the two-loop network

Diameter (inches)	Cost (Units)
1	2
2	5
3	8
4	11
6	16
8	23
10	32
12	50
14	60
16	90
18	130
20	170
22	300
24	550

(*Source: Abe and Solomatine ,1998*)

Table 2: Node data for the two loop network

Node	Demand(m^3/hr)	Ground level (m)
1(Reservoir)	-1120.00	210.00
2	100.00	150.00
3	100.00	160.00
4	120.00	155.00
5	270.00	150.00
6	330.00	165.00
7	200.00	160.00

(*Source: Abe and Solomatine, 1998*)

Table3: Summary of the optimal allocation of pipe diameters obtained from the Modified Assignment Problem Approach.

jth Pipe Diameter	ith Pipe in the Network 1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0
10	0	1	0	0	0	1	1	0
12	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
16	0	0	1	0	1	0	0	0
18	1	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
Cost Evaluation	41900							

Table 4: Comparison of the optimal pipe diameter (inches) between the previous approaches and the Modified Assignment Problem Approach.

Pipe No	Algorithm					
	CRS2	GA	ACCOL	CRS4	Best Run	MAPA
1	18	18	22	18	18	18
2	10	14	18	16	10	10
3	16	14	20	14	16	16
4	4	1	3	2	4	4
5	16	14	16	14	16	16
6	10	1	4	1	10	10
7	10	14	18	14	10	10
8	2	12	16	10	1	1
Cost(units)	422000	424000	447000	439000	419000	419000
Evaluations	10009	3381	1810	720	1372	12

Table 5: Comparing the Nodal heads (m) corresponding to optimal diameter for the previous Approaches with the Modified Assignment Problem Approach

Pipe No	Algorithm					
	CRS2	GA	ACCOL	CRS4	Best Run	MAPA
1	0.00	0.00	0.00	0.00	0.00	0.00
2	53.21	53.21	57.45	53.21	53.21	53.20
3	30.50	36.62	45.59	39.79	30.34	30.37
4	43.36	43.92	51.65	43.89	43.39	43.36
5	33.92	42.01	54.31	45.22	33.63	33.62
6	30.30	31.51	40.32	31.47	30.36	30.35
7	30.25	30.01	42.86	30.34	30.43	30.43

RESULT AND DISCUSSION

The resulting optimal allocation of the pipe diameters to the respective pipes in the network is presented in Table 3 with the associated minimum cost of 41900(units). Also a comparison between this output and some other previously reported approaches are in Table 4 while Table 5 shows the optimal nodal head associated with each of these diameters. All the algorithms have nearly the same optimal value with varying function evaluations before optimality is attained, this being an indication of the efficiency of each algorithm in question.

It can be observed from Table 3 that a binary variable 0 or 1 is assigned to each assignment as previously explained. An assignee i carries 1 when assigned to task j and zero when not used. The resulting table is actually a 14 by 14 matrix but only summarized this way for dexterity. The output has to be a square matrix for the problem to be suited for this approach. In Assignment problem, the number of assignee must be equal to the number of tasks but in this case study, the number of assignee is more than the tasks. To cater for this short coming, dummy variables had been introduced to make up. Also the modification introduced through equation 6 caters for the explanation why diameters 10 and 16 inches had to appear more than one time that is typical of this approach. This relaxation of the rule makes the summation of the decision variables along the row to be more than unity for some, unity for some and zero for the rest to obtain the desired optimal values.

Looking at Table 4, the best run algorithm reported by Abe and Solomatine (1998) had the same output with the linear programming approach (i.e. Modified Assignment Problem Approach) but with very different efficiency as MAPA solution was obtained for the two loop network within 53 seconds using MATLAB 6.5 version computing environment which is very efficient in computing on a computer with Pentium 233 MHz processor as against 7 minutes for Best Run on a computer with Pentium 100MHz processor reported by Abe and Solomatine (1998). This wide margin in time might be traced to the number of functions evaluation which is 12 for MAPA due to more number of parameters specified and 1372 for the best run. Also, linear systems are characterized by straight forward direction of search for optimum unlike the non-linear system where various options have to be sought before the optimum is obtained. This will also mean a higher storage requirement for the non-linear systems than the linear system. This same explanation is also applicable to the observations on Table 5 that gives the optimum nodal head that corresponds to these optimal diameters so as to satisfy the hydraulic requirements.

However, one can see that the optimum solution given by each of the previous algorithms and that of MAPA represent different pipe networks with varying cost and this might give the managers varieties of options in their decision making.

CONCLUSIONS

Within the context of the size of the problems that have been solved, one could readily conclude on the effectiveness of the MAPA as it produces result similar to the previously reported best run but more efficient as it possesses lower function evaluations that culminated in appreciable time saving. Also, this result will be suited for analysts in their choice of suitable algorithm when it comes to the size of the network in question. Mention must be made of the fact that the accuracy of an MAPA lies much on the proper formulation, choice of constraints and right parameter estimation. This approach of MAPA will be useful especially for the upcoming developing nations where their academics and even the industries do not have adequate access to the emerging software in this regard.

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