*LAUTECH JOURNAL OF ENGINEERING AND TECHNOLOGY 1(1) 2003: 1-5*

# GENERALIZED ANALYSIS OF THE MULTI-WELL INTERFERENCE TEST IN HYDROCARBON RESERVOIRS

**Falade, G.K. and Isehunwa, S.O.**

Department of Petroleum Engineering,
University of Ibadan, Nigeria.

*ABSTRACT*

***This study presents a generalized analytical formulation of the Multi-Well Interference test problem using the concept of the source and sink functions as a tool for solving the basic interference equations. The much broader case, featuring inner well boundary conditions with skin and storage factors at both the active and the several observation wells, is considered. The general nature of the formulation allows for the analyses of non-homogeneous reservoirs and hence the concepts of reservoir directional permeability and the related issues of reservoir principal axes of permeability were addressed. Most of the published cases of interference tests were identified as limiting cases of the generalized formulation.***

## INTRODUCTION

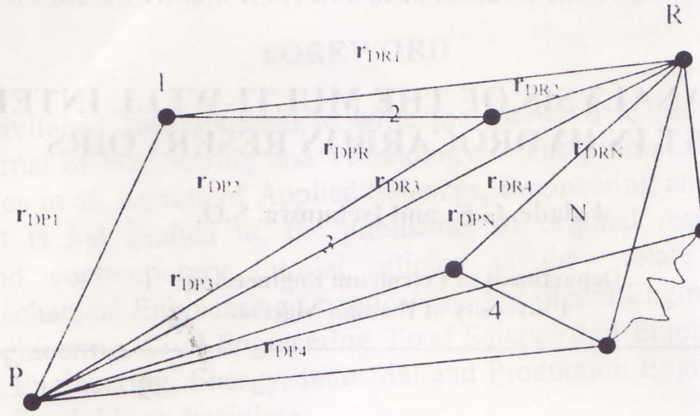
Pressure Transient Analysis has become a very important tool for reservoir characterization. The most commonly used pressure transient analysis technique is the single well test, either in form of pressure drawdown or pressure build-up tests. These tests are based on the easy assumption of homogeneous reservoirs, and are therefore limited to the evaluation of volume-averaged horizontal permeability of the reservoir zone contacted during the test. In view of the averaging process of these tests they are notoriously incapable of quantifying lateral variations in reservoir properties no matter how severe.

Multi-well interference tests, featuring an active well and one or more observation wells do, unlike the single-well tests, present powerful alternative means of using well test as a reservoir characterization tool. In this kind of interference tests, reservoir parameters that relate to inter-well properties, such as the problems of directional permeability in heterogeneous reservoirs, are addressed. Directional permeability has very strong implications in non-homogeneous reservoirs when external energy input in form of fluid injection is required to augment natural reservoir energy. In these cases, good understanding of the nature of reservoir heterogeneity, and the directional permeability are very important in order to choose the optimum fluid flood pattern orientation that would reduce the adverse effects of fluid channeling on area sweep efficiency. Similarly, in order to optimize production from horizontal wells, knowledge of directional permeability and/or the principal axes of reservoir permeability are essential so that wells are preferably drilled orthogonal to the direction of maximum permeability.

A few papers have recently been published on the evaluation of directional permeability using multi-well interference tests[1,2]; none of these has attempted a unified treatment of the problems of heterogeneity and possible directional permeability in interference well test. In this paper we will attempt to formulate the interference well test equations in a generalized form using the concept of source functions. This formulation would be sufficiently general such that cases involving homogeneous reservoirs can be considered as special cases, and therefore most of the published interference equations can be seen as limiting cases of the generalized equation.

## PROBLEM FORMULATION

We consider the case of tests with a single active well and several observation wells as shown in the schematic below.




The relevant distances between any observation well 'R' and active well 'P' are as indicated in the schematic. The pressure response at the observation well 'R' due to a production pulse at the active well 'P', is affected by pressure interference from all the other wells within its neighborhood. The level of interference due to each of these surrounding wells is normally a function of the rate at which the well is produced as well as its distance from the observation well. The pressure response function at the wellbore of the observation well can be expressed, in Laplace space, using the principle of superposition in space in a generalized interference equation given as[3]:

$$P_D(r_{DRR}, \lambda) = \omega_P G(r_{DPR}, \lambda) + \omega_{RR} G(r_{DRR}, \lambda) + \sum_{\substack{j=1 \\ j \neq R}}^{N} \omega_{Rj} G(r_{DRj}, \lambda) \quad \ldots\ldots \quad 1$$

In this equation, it is pertinent to note that the distance designation, $r_{DPP}$ and $r_{DRR}$ must each be unity since they describe the distance of the active well from the active wellbore location and the observation well from the observation wellbore location respectively. The factor $\omega_i$ represents the weighting function that reflects the level of pressure dominance of any particular adjoining well at the observation wellbore.

**Pressure Profiles at The Observation Well**

Using the above solution scheme, the pressure equations at all the 'N' observation wells, taking account of their respective wellbore Skin factor ($S_{RR}$), can be expressed as:

$$P_{DRR}(r_{DRR}, \lambda) = \omega_P G(r_{DPR}, \lambda) + \omega_{RR}[S_{RR} + G(r_{DRR}, \lambda)] + \sum_{\substack{j=1 \\ j \neq R}}^{N} \omega_{Rj} G(r_{DRj}, \lambda) \quad \ldots \quad 2$$

Equation 2, contains parameters $\omega_P$ and $\omega_{Rj}$ which can be related through the use of wellbore Storage constant ($C_{DRR}$) and the other wellbore boundary conditions to obtain:

$$\omega_P[C_{DRR}\lambda G(r_{DPR}, \lambda)] + \omega_{RR}\{1 + C_{DRR}\lambda[S_{RR} + G(r_{DRR}, \lambda)]\} + C_{DRR}\lambda \sum_{\substack{j=1 \\ j \neq R}}^{N} \omega_{Rj} G(r_{DRj}, \lambda) = \frac{\alpha_R}{\lambda} \quad \ldots \quad 3$$

$\alpha_R$ is the ratio of the fluid production rate at the observation wellbore ( j=R ) to the production rate at the active wellbore (P).

Equation 3 is a set of N indicial equations, one equation for each of the observation s in the multi-observation well test.

**Pressure Profiles at The Active Well**

Proceeding as in the observation well case, the pressure response equation at the only active well, taking cognizance of the pressure interference from all the observation wells, can be expressed in Laplace space as:

$$P_{DP}(r_{DPP}, \lambda) = \omega_P[G(r_{DPP}, \lambda) + S_P] + \sum_{j=1}^{N} \omega_{Rj} G(r_{DPj}, \lambda) \quad \ldots\ldots \quad 4$$

And the single indicial equation that relates $\omega_{Rj}$s and $\omega_P$ is given as:

$$\omega_P\{1 + C_{DP}\lambda[S_P + G(r_{DPP}, \lambda)]\} + C_{DP}\lambda \sum_{j=1}^{N} \omega_{Rj} G(r_{DPj}, \lambda) = \frac{1}{\lambda} \quad \ldots\ldots\ldots\ldots \quad 5$$

Equations 3 and 5 constitute a set of (N+1) simultaneous equations from where all the functions $\omega_p$, ($\omega_{Rj}$, j = 1,2,3….N) can be evaluated. These equations can be expressed in terms of Vector - Matrix notation as:

$$\begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,N+1} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,N+1} \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,N+1} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{N,1} & a_{N,2} & a_{N,3} & \cdots & a_{N,N+1} \\ a_{N+1,1} & a_{N+1,2} & a_{N+1,3} & \cdots & a_{N+1,N+1} \end{vmatrix} \begin{vmatrix} \omega_{R1} \\ \omega_{R2} \\ \omega_{R3} \\ \vdots \\ \omega_{RN} \\ \omega_P \end{vmatrix} = \begin{vmatrix} \alpha_1/\lambda \\ \alpha_2/\lambda \\ \alpha_3/\lambda \\ \vdots \\ \alpha_N/\lambda \\ 1/\lambda \end{vmatrix} \qquad 6$$

The larger the number of the observation wells involved in the interference test, the larger the indicial matrix equation that needs to be solved, and the more complex the analyses of the test results.
If there were only two observation wells and one active well, then N=2 and R= 1,2; and the indicial equation is reduced to a 3x3 matrix-vector equation given as:

$$\begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{vmatrix} \begin{vmatrix} \omega_{R1} \\ \omega_{R2} \\ \omega_P \end{vmatrix} = \begin{vmatrix} \alpha_1/\lambda \\ \alpha_2/\lambda \\ 1/\lambda \end{vmatrix} \qquad 7$$

where $a_{i,j}$ are defined as follows:

$$a_{1,1} = C_{DR1}\lambda( S_{R1} + G(r_{DR1}, \lambda) ) + 1 \qquad 8a$$
$$a_{1,2} = C_{DR1}\lambda G(r_{DR2}, \lambda ). \qquad 8c$$
$$a_{1,3} = C_{DR1}\lambda G(r_{DP1}, \lambda ). \qquad 8d$$
$$a_{2,1} = C_{DR2}\lambda G(r_{DR1}, \lambda ). \qquad 8e$$
$$a_{2,2} = C_{DR2}\lambda( S_{R2} + G(r_{DR2}, \lambda) ) + 1 \qquad 8f$$
$$a_{2,3} = C_{DR2}\lambda G(r_{DP2}, \lambda ). \qquad 8g$$
$$a_{3,1} = C_{DP}\lambda G(r_{DP1}, \lambda) \qquad 8h$$
$$a_{3,2} = C_{DP}\lambda G(r_{DP2}, \lambda ). \qquad 8i$$
$$a_{3,3} = C_{DP}\lambda( S_p + G(r_{DPP}, \lambda) ) + 1 \qquad 8j$$

Using equation (8), the indicial matrix equation for the three well interference test can be solved for the weighting parameters $\omega_{R1}$, $\omega_{R2}$ and $\omega_P$ in order to construct the test response equations. Even for this seemingly simple case, the test response function could, in theory, be complex and unwieldy.
By far the most commonly used interference test geometry is the two-well interference test geometry[4,5,6,7,8]. For the very simple but general case of the two-well test (one observation well and one active well), N is unity; thus N=1=R. The indicial response equation reduces to a 2x2 matrix-vector form given as:

$$\begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} \begin{vmatrix} \omega_{RR} \\ \omega_P \end{vmatrix} = \begin{vmatrix} \alpha_R/\lambda \\ 1/\lambda \end{vmatrix} \qquad 9$$

Where:

$$a_{1,1} = C_{DR1}\lambda( S_{R1} + G(r_{DR1}, \lambda) ) + 1 \qquad 10a$$
$$a_{1,2} = C_{DR1}\lambda G(r_{DP1}, \lambda ). \qquad 10b$$
$$a_{2,1} = C_{DP}\lambda G(r_{DP1}, \lambda ). \qquad 10c$$
$$a_{2,2} = C_{DP}\lambda( S_p + G(r_{DPP}, \lambda) ) + 1 \qquad 10d$$

Equation 9 can be solved for $\omega_P$ and $\omega_{RR}$ using Crammer's rule to obtain:

$$\omega_P = \frac{a_{1,1} - \alpha_1 a_{2,1}}{\lambda(a_{1,1}a_{2,2} - a_{2,1}a_{1,2})} \qquad 11a$$

$$\omega_{RR} = \frac{\alpha_1 a_{2,2} - a_{1,2}}{\lambda(a_{1,1}a_{2,2} - a_{2,1}a_{1,2})} \qquad 11b$$

Using Equation 11 in 2 for a two-well situation where N=1=R, the pressure profile at the observation well R can be expressed in the form:

$$P_{DRR}(r_{DRR},\lambda) = \frac{[1 - \alpha_R \lambda C_{DP} \lambda G(r_{DPR},\lambda)]G(r_{DPR},\lambda) + \alpha_R [C_{DP}\lambda(S_P + G(r_{DPP},\lambda)) + 1][S_R + G(r_{DRR},\lambda)]}{\Delta} \quad ...12$$

Similarly, using equation 11 in 4, we can express the pressure response equation at the active well as:

$$P_{DP}(r_{DP},\lambda) = \frac{[C_{DRR}\lambda\{S_{RR} + G(r_{DRR},\lambda)\} + 1][S_P + G(r_{DPP},\lambda)] + \alpha_R G(r_{DPR},\lambda) - C_{DRR}\lambda G(r_{DPR},\lambda)G(r_{DPR},\lambda)}{\Delta} \qquad 13$$

In equations 12 and 13, the function $\Delta$ is given as:

$$\Delta = \lambda\,(1 + C_{DP}\lambda\,[\,S_P + G(r_{DPP},\lambda)\,]\,\}\,\{1 + C_{DRR}\,\lambda\,[\,S_{RR} + G(r_{DRR},\lambda)\,]\,\} - C_{DP}C_{DR}\,[\lambda G(r_{DPR},\lambda)\,]^{2}] \qquad 14$$

**Applications**

If in a two-well interference test, for which generalised equations 12 and 13 are applicable, we assume that the observation well is shut in, as is usually the case, then the parameter $\alpha_R$ appearing in these equations becomes identically zero. Under this condition, Equations 12 representing the pressure response functions at the observation well would simplify to the following:

$$P_{DRR}(r_{DRR},\lambda) = \frac{G(r_{DPR},\lambda)}{\Delta} \qquad 15$$

While equation 13 for the active wells pressure response profile simplifies to equation 16 given as:

$$P_{DP}(r_{DP},\lambda) = \frac{[C_{DRR}\lambda\{S_{RR} + G(r_{DRR},\lambda)\} + 1][S_P + G(r_{DPP},\lambda)] - C_{DRR}\lambda G(r_{DRP},\lambda)G(r_{DRP},\lambda)}{\Delta} \qquad 16$$

The denominator of both equations 15 and 16 remains as:

$$\Delta = \lambda\,(1 + C_{DP}\lambda\,[\,S_P + G(r_{DPP},\lambda)\,]\,\}\,\{1 + C_{DRR}\,\lambda\,[\,S_{RR} + G(r_{DRR},\lambda)\,]\,\} - C_{DP}C_{DR}\,[\lambda G(r_{DPR},\lambda)\,]^{2}] \qquad 17$$

It is perhaps interesting to note the symmetry of equation 14, and by extension, the entire equation 15, in ($C_{DP}$, $S_P$) and ($C_{DRR}$, $S_{RR}$). These sets of parameters can be interchanged without changing the value or meaning of the entire pressure response function. Equation 15 is a very elegant demonstration of the mathematical proof of the 'Reciprocity Principle'[8]. This principle suggests that it would not matter which of the two wells involved in the interference test is used as the observation well, the pressure response measured would be same.

If in the two well test system, the source function $G(r_{DPR},\lambda)$ is assumed to be the instantaneous line source solution in Laplace space, these equations 15 and 16 readily simplify to the interference equations commonly published in the Petroleum Engineering literature[9,10].

## CONCLUSIONS

A new set of generalised formulations has been developed for pulse test configurations with multiple observation wells. The formulations uses the concept of source functions in Laplace space with the principle of superposition in space to account for the different locations of the various observation wells within the test domain. The new formulations allow for wellbore Skin and Storage factors to exist in either or both the active and observation wells.

One major advantage of the new model is the fact that, in a multiwell interference test, the response function for any of the observation wells is well spelt out; and can therefore be analysed independently of the other observation wells involved in the test. The traditional need to assume the line source solution in order to type-curve-match interference response data will not now be necessary. It is also important to emphasise that most of the traditional ideas and concepts already successfully developed for various forms of interference tests could find their equivalents as limiting cases of the new generalised formulations.

## REFERENCES


Falade, Gabriel K. " Pulse and Interference Test Design, Implementation and Interpretation in Upper Zakum" Zadco's Reservoir Engineering Report, August 1998.

Johnson, C. R., Greenkorn, R. A., and Woods, E. G.: "Pulse Testing: A New Method Describing Reservoir Flow Properties Between Wells", Trans. AIME 237, 1599-1604, 1966.

Swift, S. C. and Brown, L. P.: Interference Testing for Reservoir Definition: The State of The Art".' SPE-5809; SPE-Improved Oil Recovery Meeting, Tulsa, Okla., March 22-24, 1976.

Burns, William A. Jr.: "New Single-Well Test for Determining Vertical Permeability". Trans. AIME, 246, 743-752., 1969

Tongpenyai, Y. and Raghavan, R.: "The Effect of Wellbore Storage and Skin on Interference Test Data" J. Pet. Tech. pp151-160 Jan. 1981.

Prat, M. and Scott, J. B.: "Effect of Wellbore Storage on Pulse Test response", J. Pet. Tech. pp707-709, Jan. 1975

Ramey, H. J. Jr.: "Interference Analysis for Anisotropic Formations- A case History", J. Pet. Tech. pp1290-1298 Oct., 1975

Falade, Gabriel K. and Brigham, William E.: " The Analysis of Single-Well Pulse Test in a Finite-Acting Slab Reservoir". SPE-5055B; SPE-AIME 49th Annual Fall Meeting Houston, Oct. 6-9, 1974.

Ogbe, D. O. and Brigham, W. E.: "Pulse Testing With Wellbore Storage and Skin Effects", SPE Formation Evaluation, pp 29-42, March 1987.

Swift, S. C. and Brown, L. P.: "Interference Testing for Reservoir Definition: The State of The Art".' SPE-5809; SPE-Improved Oil Recovery Meeting, Tulsa, Okla., March 22-24, 1976.