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DISCRETE ELEMENT MODELLING OF THE IMPACT PARAMETERS OF A SELECTED FRUIT: MODEL VALIDATION AND EFFECT OF DAMPING

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ABSTRACT

The Discrete Element Method has been applied to numerical modelling of the impact process in fruit with a view to obtain information on damages due to impact. A DE code which incorporated a non-linear viscoelastic contact law, real containing walls and particle deformation developed from an existing code using a linear elastic contact law without containing walls was used. The code was validated with available theoretical and experimental results on impact behaviour of rubber spheres. The parametric variations with different damping coefficients were then simulated for a selected fruit dropped from a height of 10, 15 20 and 30 cm onto a hard flat surface. The parameters investigated include variation in the force, time, velocity, deformation and the peak force during contact in relation to the height of drop. The experimental and the simulated were found to be in good agreement (p>0.05) with no significant differences. The peak forces predicted by the simulation were 42.5, 85.2 and 137.7 N as compared to 40, 87 and 136 N from the experiment for spheres weighing 29.4, 95 and 190.7 g respectively. The predicted time of contact was however slightly lower in all the cases but there is no significant difference between the experimental and the predicted (p>0.05). The study with the fruits also showed good agreement with the existing data, indicated by the features of the curves, which showed that the non-linear viscoelastic contact model is a good approximation for predicting the contact behaviour of viscoelatic materials. Information on the parameters required in the selection of appropriate materials and range of parameters useful in the design of handling equipment were provided. The model is therefore a useful tool in the study of impact process for agricultural particulate (discrete) materials.

INTRODUCTION

The Discrete Element Method (DEM) is a well-established computational method for modelling the mechanical behaviour of particulate or discrete systems. The DEM differs from the Finite or Boundary Element Method (FE or BE) which treats a particulate system as a continuum. In DEM, material properties, inter-particle contact forces, displacement and other mechanical parameters are calculated over a discrete (very small) time, hence a knowledge of the interaction of the particles at the micro time scale is made possible. This therefore enables a better prediction and understanding of the behaviour over a macro time scale.

The theories governing the behaviour of agricultural particulates, contact theory and theory of elasticity were discussed by Raji (1999) and can also be found in Mohsenin (1986), Timoshenko and Goodier (1970). Generally when a purely elastic material is dropped vertically down on to a flat surface, assuming there is no external resistance force, it is expected that the material will continue bouncing. However for a non-elastic material it will bounce a number of times, loosing energy at each impact before coming to rest on the surface. On each impact the particle deform in accordance with the potential energy of fall.

In fruit handling, most of the damages are in form of bruising occurring as a result of impacts against a variety of surfaces and particularly during impacts with other fruits. This type of damage is a major cause of quality loss for fresh fruit (Siyami *et al* 1988, Studman, 1990). The deformation at impact has been used to estimate damage to some agricultural materials by many researchers. The damage and stress distribution on impacting fruits under static and dynamic loads have been studied extensively using various experimental methods resulting in a number of published parametric results on the behaviour of fruits. These include impact of fruit on a wide range of hard or padded surfaces (Schoorl and Holt, 1980; Diener *et al* 1979); bruising damage in apple-to-apple impact (Pang *et al,* 1994 Lichtensteiger *et al,* 1988) and impact damage on other fruits and contact pressure measurement (Mathew and Hyde, 1997; McGlone *et al,* 1997a and b; Herold *et al*, 2001).

Studies have also been carried out using instrumented spheres. This is a physical artificial sphere that captures each impact it experiences through an electronic sensor placed in it (Tennes *et al* 1990; Hyde *et al* 1992). Pang *et al* (1994) however observed that there has been considerable difficulty in relating the output to the level of fruit bruising.

Prediction models have also been used in bruise damage studies. The models results are usually compared with published experimental work or with results from experimental works used as the basis for the model development. (Siyami *et al,* 1988; Chen and Yazdani, 1991). The problem usually associated with these models is inexact assumptions of material behaviour leading to the use of approximate theories. These predictive models are based on the acceleration history and mass of the object. The contact theory considering the type of contact surfaces is very important in impact damage. The DEM is a method that considers both the acceleration history, mass and contact theory. The possibility of monitoring the interaction at very minute/small time interval is also an advantage over other forms of modelling methods.

Lichtensteiger *et al* (1988) measured the impact force on tomatoes and some non-agricultural materials (e.g. rubber) dropped on a very rigid and heavy base. The force data at equal time intervals were recorded via a force transducer connected to an oscilloscope and recorded or saved on a microcomputer. A sampling rate of 2.10μs per point, which gave adequate precision, was used for defining the force-time curve, and tests were repeated at a slower sampling rate of 100-500μs per point to obtain the time between the first and second bounces. Using the force-time information during contacts simple equations based on Newton's law of motion and finite difference approximation were developed to determine the velocity, acceleration and displacement during contact on a plate placed on a very heavy and rigid surface. The coefficient of restitution and energy dissipated during impact were also determined. The researchers made the following assumptions during the development of mathematical model for the latter parameters; the products are spherical, the centre of mass and the centre of the sphere remained coincident during contact, all the mass was subjected to the same acceleration and vibrations were negligible. One very good advantage of modelling is that it can eliminate or reduce the excessive time spent on experimental work and make the prediction of any process very easy.

The DEM has evolved from various disciplines such as geomechanics, physics and structural engineering so is its application on impact has been widely spread over a number of engineering materials. Impact studies using DEM was applied on rocks and sands particles by Nicott *et al* (2001) who studied the behaviour of a restraining nets for rockfalls in embankments. The DEM also found application in the pharmaceutical processes with the studies of Kafui and Thornton (1993), Thornton *et al (1996)* and Ning *et al* (1997) on fragmentation and impact breakage of agglomerates.

DEM impact application has also been used to study the damage resulting from impact and dynamic loads during transportation of fruits packed in containers on a truck bed. (Rong, 1993). The effects of packing arrangement, stack height and the road roughness on the damages to the fruit were studied. The result of the variation of the forces in the layers of the fruit which are in accord with experimental results of Holt *et al,* (1985) were used to recommend optimum packing arrangement. Schembri and Harris (1996) also applied the DEM to the study of 2-D fracture of a cellular material (sugar cane) under impact load. The failure of a transverse section of the cane with storage cells represented as hexagonal elements was obtained by observing and comparing lines along broken contact points with those from an experimental set up. Both produced Y-shaped failure pattern very similar in dimension.

It is pertinent to note that emphasis has been laid on the impact breakage of cluster of particles but drop impact of objects is a very common process in agricultural materials handling especially during harvesting and in food processing industries. An understanding of the process is therefore as important as other impact processes. There is therefore a need to extend the numerical studies to impact drop process which has been extensively studied experimentally as stated earlier. An understanding of the process will reduce losses associated with bruising which is a major cause of quality loss for fresh fruit usually recorded in fruit handling. It will also provide insight into parameters that are necessary in machinery and process design.

The objective of this study was therefore to study the behaviour of a fruit during impact using the DE model to determine the variations in some impact parameters with a view to understanding the impact process and making predictions for material selection for fruit handling.

THEORETICAL AND MODEL DEVELOPMENT

The DEM is an explicit numerical scheme in which the interaction of particles or bodies within a system is monitored contact by contact and the motion modelled particle by particle. The process involves solution of the equation of motion on each particle over very small discrete time step (Δt) using an explicit time numerical finite difference approximation (FDA). The resultant of the component forces; contact (F_c), gravity (F_g) and any other force in play (F_e) on the particle with mass *m* in contact with *N* other particles produces acceleration. Therefore

$$m\frac{d^2x}{dt^2} = F_T = \sum_{i=1}^{N} F_{c_i} + F_g + F_d + F_e \qquad 1$$

This is integrated using the FDA leapfrog integration scheme to obtain the velocity and displacement in order to determine the new particle position ($x_{t+\Delta t}$) after the time step.

$$x_{t+\Delta t} = x_t + [\dot{x}]_{t+\frac{1}{2}\Delta t}\,\Delta t$$

$$= x_t + \left[\dot{x}_{t-\frac{1}{2}\Delta t} + (\ddot{x})_t\,\Delta t\right]\Delta t \qquad 2$$

The contact forces between a particle and its immediate neighbours are calculated using appropriate force-displacement laws. In DEM, particles are usually modelled as spheres for ease of computation due to the availability of established relationship between two contacting spheres. In this study the Hertz theory which gives the forces between two contacting elastic convex surfaces was used. The theory gives is a nonlinear relationship in form of the linear F = kx. The use of a pre-determine contact stiffness coefficient *k* was eliminated and replaced with a dynamic/variable contact stiffness and the displacement (x) as the approach of the centers of the two bodies (α). The contact stiffness is dependent on: the strength property (*K*, which is dependent on the elastic modulus and Poisson ratio of the contacting bodies), the geometry (R, dependent on the radii of the two spherical bodies). Hence

$$F_c = K^{-1} R^{-\frac{1}{2}} \alpha^{\frac{3}{2}} \qquad 3$$

The theory has also been extended to the contact between concave/flat and convex bodies. Therefore the computation of the contact between spherical objects and container wall was made easy. Further details on the theory can be obtained in Timoshenko and Goodier (1970). The extension by Raji (1999) to other bodies as well as its adaptation to include viscous element for viscoelastic materials is presented in Raji and Favier (1998) and Favier and Raji (2002). According to Tsuji *et al* (1992) and Zhang and Whitten (1996) viscoelasticity is more realistic for agricultural particulates than the more commonly used linear elastic models. The works were corroborated by the investigation of Zhang and Whitten (1996) on different methods of calculating contact forces especially in DE. They found that the non-linear formula of Tsuji *et al* (1992) showed that the force changes sign (returns to zero indicating particle separation) before the displacement returns to zero (residual deformation before physical particle separation) which is a more realistic result than the more commonly used linear approach. The theoretical solutions were confirmed and found to be closer to experimental results through curve fittings.

Raji (1999) therefore developed a model by incorporating the theories presented above into the DE principle to study the interaction of particulate materials. The model based on an earlier model by Ng (1989) for particulate soil and rock materials was modified to handle agricultural materials. The model as reported by Raji and Favier (1998), Favier *et al* (1999) incorporated a non-linear deformation-dependent condition for viscoelastic materials into an elastic model for spherical materials as stated with algorithms to handle change in shape of the particles during loading, a condition which is normally neglected in DEM.

The model also incorporated real containing walls modelled as flat surfaces. In impact studies only one wall or flat surface on which the particle(s) will impact is necessary and in drop impact only the gravity force is in play before impact except where drag force is very necessary. For a body falling on a surface from a height the initial total energy is equal to the potential energy which is proportional to the mass and drop height and it is represented by the area under the force-deformation curve. The energy on the object immediately after contact when the particle escapes from the surface is equal to the kinetic energy remaining on the object after contact. This is equal to the area behind the rebound contact recovery curve in a F-D plot (Raji, 1999; Raji and Favier, 2002). The difference gives the energy dissipated

Method

The Raji (1999) model was used to predict the behaviour of a single spherical rubber ball dropped from 10, 15, 20, and 30 cm on to a hard flat surface. A comprehensive description of the mode of operation of the model can be found in Raji (1999) and Raji and Favier (1998). In order to validate the results obtained from the modelling for impact process, available experimental and theoretical results (Lichtensteiger *et al*, 1988; Zhang and Whitten, 1996) on impact studies for spherical rubber balls using drop tests were selected for comparison. The results of these studies were used for comparison because the instrumentation and the precision of the instruments used in data recording makes the results more reliable than other available results. The parameters determined during the experiment also covered a wide range of parameters required for a better understanding of impact processes. Table 1 shows the physical properties of the rubber balls used in the experiments and simulations.

Table 1: Data used in simulation as obtained from Lichtensteiger *et al* (1988) for ADRUB balls (Rigidity Modulus: 3.00 MPa, Friction coefficient: 0.5, Poisson ratio: 0.31).

Diameter (cm)	Mass (g)	Coefficient of restitution	Time step (µs)
3.76	29.4	0.636	4.1
5.47	95.0	0.639	4.4
7.04	190.7	0.629	4.6

Simulation

Numerical simulations of the impact of rubber balls and apples were performed using the new computer programme with the rheological model for viscoelastic materials as explained above i.e. the Raji (1999) model. The properties of the materials from the experiment of Lichtensteiger *et al* (1988) (Table 1) were used in the computer simulation for rubber balls while those of apple were obtained from the literature (Mohsenin *et al*, 1978; Mohsenin 1986). The rigidity modulus of the rubber material is 3.0 MPa. In the simulation, similar to the experiment, the heavy and rigid base was simulated as a flat wall with infinite radius of curvature and high strength properties compared to the spherical material impacting upon it. Each simulation involved creating a spherical particle with the appropriate physical properties and geometry at the desired height of fall (10, 15, 20 and 30 cm). Impact data (force, velocity, and displacement with time) during the first impact and the peak values during the other impact were recorded for every cycle (single time step) until the particle came to rest. It is to be noted however, that the data in the experimental report in Lichtensteiger *et al* (1988) did not extend as far as particle equilibrium (coming to rest).

RESULTS AND DISCUSSION

The simulation results obtained for the ADRUB rubber ball were compared with the experimental results reported by Lichtensteiger *et al* (1988) as a form of validation of the contact model and suitability of the DEM for the experiments discussed above. Analysis of the agreement of the experimental and simulation results were done statistically using both analysis of variance and studentised-t tests to confirm significance.

Validation with Rubber Ball

The force-time curves during the first impact and the effect of mass obtained from impact of three rubber balls of different weight dropped from a height of 10cm on a rigid plate are as shown in Figure 1. The simulation results using the properties and geometry presented in Table 1 were compared with the experimental result of Lichtensteiger *et al* (1988) on the same material and set up. A critical time step for the simulations range between 4.1 and 4.6 μs and these coincidentally are within the range of 2-10 μs used in data recording in the experiment. Simulation results showing the peak force and time of contact for the first impact when dropped from heights 10, 15, 20 and 30 cm plotted with those of Lichtensteigers' are presented in Figures 4 and 5. The deformation and velocity obtained with time and velocity-deformation curves for the first impact in the 10 cm drop height are presented in Figures 4 - 6 while the force-deformation curve for each of the test is as shown in Figure 7 although these are not reported by Lichtensteiger *et al* (1988).

Good agreement was obtained between the simulation results and Lichtensteiger *et al* (1988) data with analysis showing that there was no significant difference ($p>0.05$) in the experimental and simulation data plotted at different time interval during impact in Figure 1. The peak forces predicted by the simulation were 42.5, 85.2 and 137.7 N as compared to 40, 87, and 136 N from the experiment for balls with weights of 29.4, 95 and 190.7 g respectively. This trend is also similar in the simulation of drop impact for the heaviest ball, (191 g), from different heights (Figure 2) and the predicted time of contact is slightly lower in all cases (Figure 3). Analysis of both (peak force and time of contact) showed no significant difference between the experimental and predicted ($p>0.05$).

Additional parameters not reported by Lichtensteigers *et al*, (1988); deformation versus time, velocity versus time and the force-deformation curves are shown in Figures 4 – 7. From Figure 5, the escape velocity for each ball is less than the contact velocity since rubber is not a purely elastic material and energy was dissipated through damping. The contact velocities (1.5 m/s) are equal as all the balls were dropped from the same height while the escape velocities were very close (no significant difference, $p>0.05$) for all three cases. This is because their coefficients of restitution are very similar but the time of contact differed due to their differences in size and weight. The ball regained its deformed portion gradually after reaching the maximum while the velocity started increasing, corresponding to unloading (Figure 6). The effect of the damping is shown with a sharp rise in the force-deformation curves in Figure 7 at the initial region where some of the energy at impact was absorbed. The force-deformation curve is therefore a loop with the area within the loop representing the energy dissipated as will be seen later.

Validation with Apples

The results reported above show that the non-linear viscoelastic contact model is a good approximation for predicting the contact behaviour of viscoelatic materials. Therefore this model was used for further studies on predicting the behaviour during impact for a viscoelastic material. The data presented in Table 2 are properties of apple fruit used for simulation of impact tests of the fruit when dropped from a height of 20 cm.

Table 2. Properties of apple used in simulation (Mohsenin, 1986)

	Diameter (cm)	Density (kg/m^3)	Elastic modulus (Mpa)	Poisson ratio	Friction coefficient Wall	Friction coefficient Ball
Range	5.00-8.00	700-900	1-10	-	0.32-0.44	
Selected	7.00	801	4.66	0.31	0.35	0.35

Effect of Damping Ratio

The results of simulations performed to study the effect of damping coefficient on the behaviour of a fruit dropped on a heavy rigid surface are shown in Figs. 8 - 13. These show the force-time, deformation and velocity time curves obtained with a range of damping coefficients during the first contact. From Figure 8 there is no direct relationship between the peak force and the damping ratio. Within the range used the peak force dropped initially with increasing damping coefficient but later rose with increasing damping coefficient. This effect is a phenomenon related to the velocity and deformation variations on which the damping force is dependent (Raji, 1999). The time of contact on the other hand showed a direct proportionality.

The deformation curve (Figure 9) is perfectly symmetrical for the elastic condition (no damping) while the skewness increased with increasing damping resulting from longer time for recovery process. This is in agreement with the theoretical results of Zhang and Whitten (1996) as discussed above. Since the details of the properties of apple are obtained from the literature validation was on the basis of agreement of the features of the curves with the experimental and general solutions for viscoelastic materials obtained by Zhang and Whitten (1996). These are in turn in agreement with the available results from experimental work on viscoelastic particulate agricultural materials (Lichtensteiger *et al*, 1988; Fluck and Ahmed, 1973). The deformation determined represents the deformation in apple only if the impact surface is harder than the fruit. If the stiffness of the contacting objects is reversed with the apple being the hard material then the deformation represents purely the deformation of the soft flat surface.

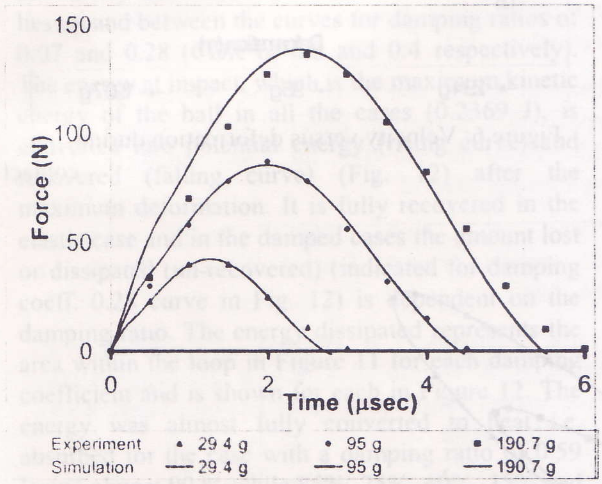

Figure 1: Force-time for rubber balls falling from a height of 10cm (Data points show the results of Lichtensteiger *et al*, 1988)

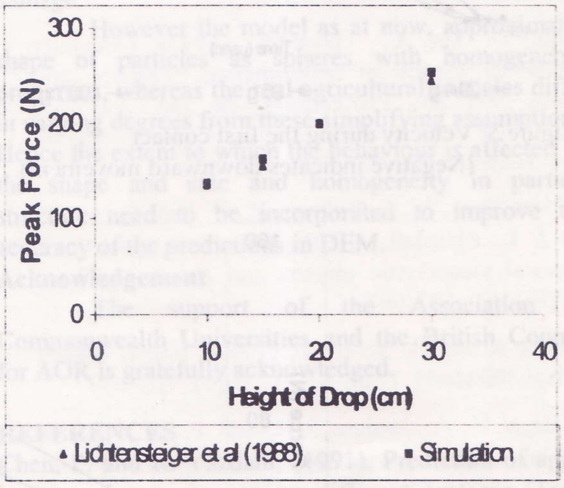

Figure 2: Peak force for different height of fall for 190.7 g rubber ball

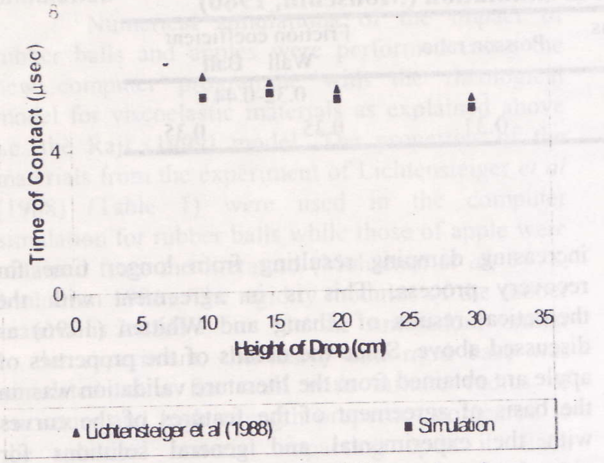

Figure 3: Time of contact for different height of fall for 190.7 g rubber ball

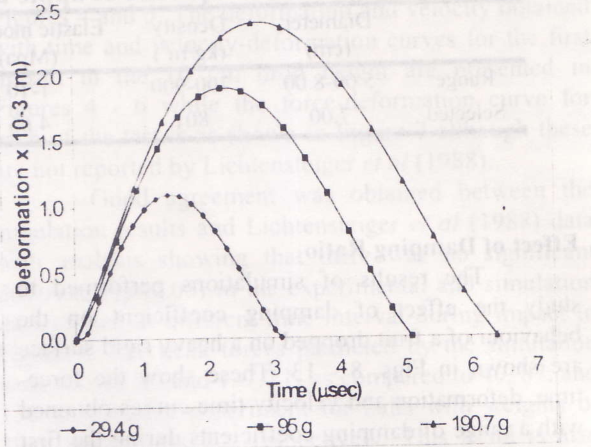

Figure 4: Deformation predicted during the first contact.

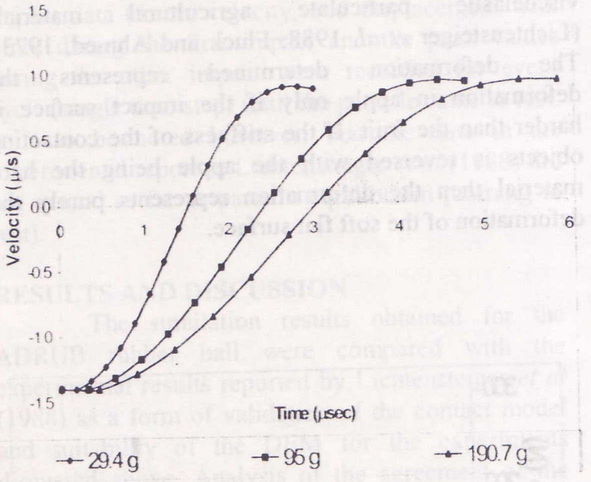

Figure 5: Velocity during the first contact (Negative indicates downward movement).

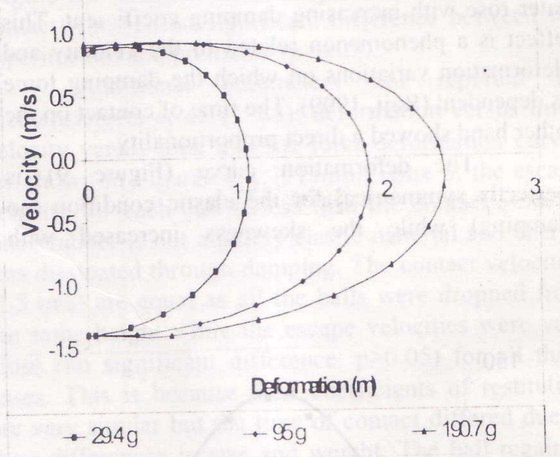

Figure 6: Velocity versus deformation during contact

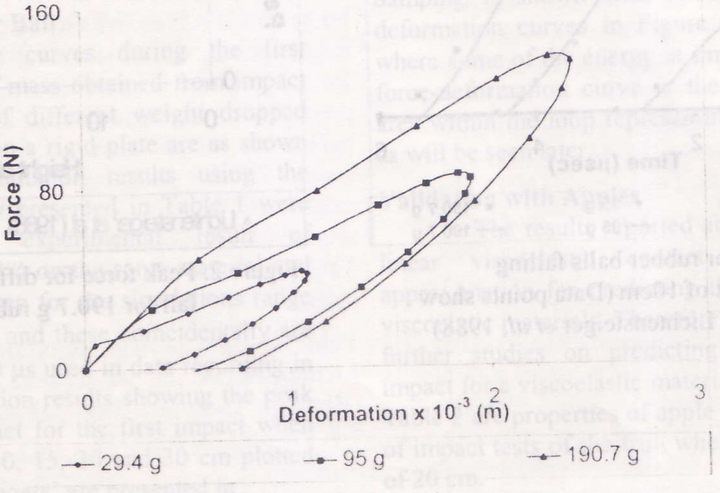

Figure 7: Force-Deformation curves

The deformation therefore in this case can give the thickness of the material that could be used as a safe padding surface for apples dropped from the given height. Report on further studies on this aspect will be presented in another paper. The information from such tests could be used in the design and material specification for handling machinery components such as conveying belt and packing materials during transportation.

Figures 11 and 12 show the force-deformation curves and the energy during contact respectively. The energy absorbed (Fig. 12) shown by the sharp rise (Fig. 11) in the initial slope on the curves increased with increasing damping coefficient. The slope of the damped region representing the energy absorption is dependent on the material properties, i.e. the degree of homogeneity of the object (Lichtensteiger *et al*, 1988) and the time step between data points. The steep slope or sharpness of the damped region (almost parallel to the y-axis) when compared to that of Zhang and Whitten (1996) is due to the softer material used (apple) and the time step used in data recording (micro seconds). Lichtensteiger *et al* (1988) found that this slope is more pronounced (almost parallel to the vertical axis) for soft objects with tough skin which they referred to as the skin effect but this is not modelled in this study

The coefficient of restitution (COR) of apple reportedly lies between 0.4 and 0.7 depending on variety and maturity. Therefore the behaviour lies on and between the curves for damping ratios of 0.07 and 0.28 (COR of 0.8 and 0.4 respectively). The energy at impact, which is the maximum kinetic energy of the ball in all the cases (0.2369 J), is converted into potential energy (rising curve) and recovered (falling curve) (Fig. 12) after the maximum deformation. It is fully recovered in the elastic case and in the damped cases the amount lost or dissipated (un-recovered) (indicated for damping coeff. 0.28 curve in Fig. 12) is dependent on the damping ratio. The energy dissipated represents the area within the loop in Figure 11 for each damping coefficient and is shown for each in Figure 12. The energy was almost fully converted to heat i.e. absorbed for the case with a damping ratio of 0.59 losing almost 90 % while 85%, 75%, 60%, 35% and 0% were lost for damping ratios of 0.46, 0.28, 0.16, 0.07 and 0.0 respectively. For all the cases with high damping the peak force and the peak deformation occurred at different times. The displacement-time curve (Figure 13) showing the movement during the entire process for each case until the particle came to rest, showed that at damping ratio of 0.59, the system was almost fully damped. This may not be applicable to apple, as it may not have a damping ratio of 0.59 except when rotten but served to illustrate the effect of damping. The range applicable to apple fruit as stated earlier is between 0.07 and 0.28. It is expected that damping ratio greater than 0.59 will result in full damping with the particle getting stuck to the surface.

CONCLUSIONS

The DEM with its capability for incorporating characteristic theories and constitutive equations for various operations on particulate media has been successfully used to model the impact process in a selected agricultural material. The extension of DEM to impact analysis was validated with data on ADRUB balls, and applied to apples to provide some fundamental insights into the impact of particulate materials, without recourse to destructive real-life experimentation. Although only selected damping values were studied in impact simulation of the apple fruit on a hard surface the agreement in the features of the curve representing the behaviour of viscoelastic materials with available results showed that using the appropriate parameters, the technique is a useful tool in the prediction of damages to agricultural particulates under impact load. The model is therefore a useful tool and a promising approach for obtaining necessary data for design of some components of handling machinery in direct contact with product in order to reduce impact damage.

However the model as at now, approximated shape of particles as spheres with homogeneous properties, whereas the real agricultural particles differ in varying degrees from these simplifying assumptions. Hence the extent to which the behaviour is affected by the shape and size and homogeneity in particle structure need to be incorporated to improve the accuracy of the predictions in DEM.

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REFERENCES

Chen, P. and R. Yazdani; (1991). Prediction of apple bruising due to impact on different surfaces. Trans ASAE, 34(3): 956-961.

Diener, R. G; Elliot, K; Nessebroad, P. E; Engle, N; Adams, R. E. and S. H. Blizzard (1979). Bruise Energy of Peaches and Apples. Trans. ASAE 22(2): 287 – 290.

Favier, J. F; Abbaspour-Fard, M. H; Kremmer, M. and A. O. Raji; (1999). Shape representation of axi-symmetrical, non-spherical particles in discrete element simulation using multi-element model particles. Engineering Computations: Int. Jour. for Computer Aided Engineering and Software 16(4): 467-480.

Favier, J. F. and A. O. Raji; (2002). Discrete element modelling of deformation in viscoelastic particulates under bulk compressive loading: Theory, Model development and Validation. Journal of Food Engineering. Communicated

Fluck, R. C. and E. M. Ahmed; (1973). Impact tests of fruits and vegetables. Trans ASAE, 16(4): 660-665.

Herold b; Geyer m. and C. J. Studman; (2001). Fruit contact pressure distribution equipment. Computers and Electronics in Agriculture, 32(3): 167 179

Hyde, G. M; Brown, G.K; Timm, E. J. and W. Zhang; (1992). Instrumented sphere evaluation of potato line impacts. Trans ASAE 35(1): 65-69.

Holt, J. E; Schoorl, D. and C. Lucas (1983). Prediction of Bruising in Impacted Multilayered Apple Packs. Trans ASAE 24(1): 242 – 247.

Holt, G.E; Schoorl, D. and C. Lucas; (1985). A theoretical and experimental analysis of the effect of suspension and road profile on bruising in multilayered apple packs. Journal of Agricultural Engineering Research, Vol. 31.

Kafui, D. and C. Thornton; (1993) Computer simulated impact of agglomerate. Powder and Grains 93, the Proceedings of the 2nd Int. Conference on Micromechanics of Granular Media. A. A. Balkema, pp. 401-406.

Lichtensteiger, M. J; Holmes, R. G; Hamdy, M. Y. and J. L. Blaisdell; (1988). Impact parameters of spherical viscoelastic objects and tomatoes. Trans ASAE 31(2): 595-602.

Mathew R and G. M. Hyde; (1997). Potato Impact damage threshold. Trans ASAE, 40(3): 705-709

McGlone V. A; Jordan, R.B. and P. N. Schaare; (1997a). Obtaining mass from fruit Impact response. Trans ASAE: 40(5): 1417 – 1419.

McGlone V. A; Jordan, R.B. and P. N. Schaare; (1997b). Mass and drop-height influence on Kiwifruit firmness by impact force. Trans ASAE, 40(5): 1421 – 1428.

Mohsenin, N. N; (1986). Physical properties of plant and animal materials. 2nd ed. Gordon & Breach Science, Publ: New York.

Mohsenin, N. N; Jindal, V. K. and A.N. Manor; (1978) Mechanics of impact of a falling fruit on a cushioned surface. Trans ASAE 21(5): 594 600.

Ng, T. T. (1989). Numerical simulation of granular soil under monotonic loading: A particle mechanics approach. Ph.D Thesis, Renselvier Polythechnic Institute, New York.

Nicott, F; Cambon, B. and G Mazollani (2001). Design of a rockfall restraining net using discrete element modelling. Rock Mechanics Engineering. 34(2): 99 118.Ning , Z; Boerefijn, R; Ghadiri, M. and C. Thornton (1997b). Discrete element simulation of impact breakage of lactose agglomerates. Advanced Powder Technology, Vol.1:15-37.

Pang, D. W; Studman, C. J. and N. H. Banks. (1994). Apple Threshold for an Instrumented Sphere. Trans ASAE 37(3): 893 – 897.

Raji, A. O (1999). Discrete element modelling of the deformation of bulk agricultural particulates. Unpblished Ph.D Thesis, University of Newcastle-upon-Tyne.

Raji, A. O. and J. F. Favier; (1998). Discrete element modelling of deformation in particulate agricultural materials under bulk compressive loading. Paper No. 98-F-045. International Conference on Agricultural Engineering, Oslo.

Raji, A O. and J. F. Favier; (2002). Discrete Element Modelling of the Impact Parameters of a Selected Fruit: Application. Journal of Applied Science and Technology, 1(2). In press.

Schembri, M. G. and H. D. Harris; (1996). Modelling impact on a biological material (sugar cane) using DEM. Paper no. 96F-072, Int. Conf. On Agricultural Engineering, Madrid.

Schoorl and Holt; (1980) Bruise resistance measurements in apple. Journal of Texture Studies 11: 389 -394

Siyami, S; Brown, G. K; Burgess, G. J; Gerrish, J. B; Tennes, B. R; Burton, C. L. and R. H. Zapp; (1988). Apple impact bruise prediction models. Trans ASAE 31(4): 1039-1046.

Studman, C. J; (1990). Apple handling damage in New Zealand. ASAE Paper No 90-6079. St Joseph, Michigan.

Tennes, B. R; Zapp, H. R; Marshall D. E. and P. R. Armstrong; (1990). Apple handling impacts data acquisition and analysis with an instrumented sphere. Journal of Agric. Engineering research, 47: 269 -276

Thornton. C; Yin K. K. and M. J. Adams; (1996). Numerical solution of the impact fracture and fragmentation of agglomerates. J. Physics D: Applied Physics 29: 424-435.

Timoshenko, S. P and J. N. Goodier; (1970). Theory of Elasticity. Third Edition. McGraw Hill, Tokyo.

Tsuji, Y; Tanaka, T. and T. Ishida; (1992). Langrangian numerical simulation of plug flow of cohesionless particle in a horizontal pipe. Powder Technology. 71 239-250.

Zhang, D. and W J. Whitten; (1996). The calculation of contact forces between particle using spring and damping models. Powder Technology, **88**: 59-64.