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OPTIMAL OPERATION OF OPA DAM - RESERVOIR WATER SUPPLY SYSTEM

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ABSTRACT

Dynamic programming technique was used in the simulation of Obafemi Awolowo University Campus water supply system to generate the maximum cumulative returns from sales of water to the people around for a seasonal planning period of a year. Major parameters necessary in the optimization of the reservoir system viz: reservoir size; rates of evaporation, siltation and seepage; streamflow characteristics and scheduled release of water, were considered as constraints. The cost functions expressed in Monetary Unit (MU) were derived for the sales of water. The objective function was optimized taking into consideration the probabilistic inflow of water into the reservoir. The maximum cumulative returns for the various combinations of: the state of the reservoir at the beginning of the planning period (S1); the inflow into the reservoir during the rainy season (I1) and the inflow into the reservoir during the dry season (I2), were computed and analysed, with the optimal policies for the various possible combinations obtained.

The maximum cumulative return from sales of water from the computer simulation result was found to be 8,868,570MU. This occurred when there was peak seasonal inflow into the reservoir and a decision to release 3,878,935 m³ of water, made during the planning period.

Keywords: Dam reservoir water supply system, Major parameters, Monetary Unit (MU), Optimal policy, Reservoir system.

INTRODUCTION

Opa river dam, a residual earth embankment, is located on the western side of Road 1, just North of the main pumping station of Obafemi Awolowo University (O.A.U.) campus, Ile-Ife in Osun State of Nigeria. The construction of the dam in the early seventies on the Opa River by the University was to be able to supply the campus adequate potable water. The water supply situation in the campus was good until around mid-eighties when dwindling available water resources, coupled with population explosion, made the situation to deteriorate. The University came up in 1988 with a plan to sell water to some categories of people to generate funds to sustain its waterworks. However, there was increase in water demand and the number of subscribers getting larger. The following issues became pertinent: whether the sales of too much water will not jeopardize the community's water supply in the event of little flows; and whether a commitment to supplying the University community adequately as well as generating maximum returns from sales of water could be met.

This paper therefore tackles the issue of how much water the Opa waterworks should release each season in order to maximize its returns from sales of water, without jeopardizing the reservoir operations. The paper formulates a reservoir management model based on probabilistic values of streamflow, employs dynamic programming technique to solve the model and puts the model to work by finding optimal set of rates and optimal series of water releases for the planning periods.

Mathematical optimization techniques have been successfully applied with the aid of digital computers to a wide variety of science and engineering problems. Typical problems in the field of water resources have been solved and optimization techniques presented for planning, design and management of complex water resource systems involving thousands of decision variables and constraints (Bower, Hufschidt and Reedy, 1962; Dorfman, 1962; Ladson, 1970; Beard, 1972; Miktell, 1977; Turgeon, 1980; and John, 1998). Direct application of dynamic programming approach to solving water resource systems as formulated by

Bellman and Buras is reported by Meredith et.al, (1973), where the popularity and success of the technique was attributed to the fact that the nonlinear and stochastic feature which characterize large number of water resources system could be translated into a dynamic programming formulation. Also, the advantage of effectively decomposing highly complex problems with large number of variables into a series of subproblems, which are solved recursively, has been explored by Christensen and Soliman (1986). Several approximation techniques to cater for problems necessitated by large numbers of computer programming time steps through the successive approximation, incremental dynamic programming and corridoring techniques have been proved by Jamshidi and Mohseni (1976), to be successful and reliable.

METHODS

Problem Formulation.

In formulating the objective function for maximizing returns from sales of water, some factors were considered. These factors include: the estimation of the quantity of water required per season by the university community; estimation of the ratio of the quantity of water sold for thirty monetary units to the staff of Obafemi Awolowo University to that sold at thirty-five monetary units for personalities in Ife town and environs; the unit cost of damage from flood overflow when the reservoir capacity is exceeded; rate of evaporation, seepage and siltation; capacity of impounding reservoir and probabilistic inflow into the reservoir. The seasonal time step has been fixed for six months May to October for the rainy season and November to April for the dry season.

The Objective Function

The objective function for the system under consideration is:

$Z_3[T_3] = Z_1[T_1] + Z_2[T_2]$ ... (1)

In which Z_1, Z_2 and Z_3 are cost functions expressed in Monetary Unit (MU)

Z_3 = maximum cumulative return from sales of water obtainable for the time period from T_1 to T_3.

Z_1 = maximum return that could be obtained from sales of water for the period from T_1 to T_2.

Z_2 = maximum return from sales of water for the period from T_2 to T_3.

T_1 = period at the beginning of the raining season, and hence the beginning of the design period.

T_2 = period at the end of the raining season, coinciding with the beginning of the dry season.

T_3 = period at the end of the dry season and hence the end of the design period.

Expressing the cost functions individually, we have:

$Z_1 = 4.44\alpha\ (x_1 - n) + 5.19\beta\ (x_1 - n) - 0.10w_1$ ... (2)

$Z_2 = 4.44\alpha\ (x_2 - n) + 5.19\beta\ (x_2 - n) - 0.10w_2$ ... (3)

Substituting equations (2) and (3) into equation (1), we have,

$$Z_3 = \sum_{i=1}^{N} 4.44\alpha\ (x_i - n) + 5.19\beta\ (x_i - n) - 0.10w_i \quad (4)$$

In which:

α = estimated fraction of water sold to O.A.U. staff.

β = estimated fraction of water sold to personalities in Ile-Ife town and environs.

x_i = quantity of water scheduled for release during system time T_i to T_{i+1}

n = estimated quantity of water reserved cost-free for the use of O.A.U. Community during system time T_i to T_{i+1}.

W_i = flood overflow during system time T_i to T_{i+1}.

i = period index i = 1, 2

N = maximal length of the planning period (one year)

Constraints on the System

The system constraints employed in this optimization problem are:

$S_{i+1} = S_i - X_i + I_i - F_i - E_i - P_i - W_i$ ... (5)

$JSM = 2{,}797{,}029$ ... (6)

$0 \leq S_i \leq 2{,}797{,}029$ ... (7)

$1{,}000{,}000 \leq x_i \leq S_i + I_i - E_i - P_i - F_i$ ... (8)

$S_1 + I_1 \geq 1{,}000{,}000 + E_1 + P_1 + F_1$ ... (9)

$S_2 + I_2 \geq 1{,}000{,}000 + E_2 + P_2 + F_2$ ... (10)

$S_1 + I_1 + I_2 \geq 2{,}000{,}000 + E_1 + P_1 + F_1 + E_2 + P_2 + F_2 + \ldots$ (11)

$S_3 \geq 1{,}000{,}000 + E_2 + P_2$ ... (12)

$2{,}322 \leq F_i \leq 4644$ ... (13)

$376{,}339 \leq E_i \leq 514{,}871$ ... (14)

$444 \leq P_i$ ... (15)

$W_i \geq 0$ ... (16)

Where:

S_{i+1} = state of the reservoir at the end of period T_{i+1}

JSM = maximum capacity of impounding reservoir

S_i = state of the reservoir at period T_i

I_i = probabilistic inflow into the reservoir during system time T_i to T_{i+1}

F_i = rate of siltation for time T_i to T_{i+1}

E_i = rate of evaporation for time T_i to T_{i+1}

P_i = rate of seepage for time T_i to T_{i+1}

W_i = flood overflow during system time T_i to T_{i+1}

S_3 = reservoir level at the end of the planning

X_i = scheduled release of water during period T_i to T_{i+1}

Tables 1 and 2 present the input variables and the system constraint-matrix respectively, for the reservoir optimization model.

Governing Equations

The mathematical solution technique employed in solving the optimization problem emanated from the nature of the objective function and the governing constraints. The cost function, Z_3, which represents the maximum cumulative return if the system state, S_3, at time T_3 is given by:

$$\text{Maximize} \Rightarrow Z_3 = \sum_{i=1}^{2} (4.72\ (X_i - 1{,}000{,}000) - 0.10W_i) \quad \ldots \quad (17)$$

Subject to:

$$S_{i+1} = S_i - X_i + I_i - F_i - E_i - P_i - W_i \quad \ldots \quad (18)$$

Table 1: Input Variables for the Reservoir Optimization Problem.

S/N	Input variables	Numerical values per season (m³)
1	F_i	2322
2	E_i	376339
3	P_i	444
4	JSM	2797029
5	F_{i+1}	4644
6	E_{i+1}	514871
7	P_{i+1}	444

Table 2: System Constraint-Matrix for the Reservoir State Model.

S/N	Variables as multiples of JSM				X_i	X_{i+1}
	S_i	I_i	I_{i+1}	S_{i+1}	(m³)	(m³)
1.	0.00	0.25	0.00	0.00	0320152.25	0179296.25
2.	0.25	0.50	0.25	0.25	1019409.50	0878553.50
3.	0.50	0.75	0.50	0.50	1718666.75	1577810.75
4.	0.75	0.00	0.00	0.75	2417924.00	2277068.00
5.	1.00	0.00	0.00	1.00	3117181.25	2976325.25
6.	0.00	0.00	0.00	0.00	3816438.52	3675582.50
7.	0.00	0.00	0.00	0.00	4515695.75	0000000.00

$0 \leq S_i \leq 2{,}797{,}029$ .. (19)

$0 \leq S_{i+1} \leq 2{,}797{,}029$ .. (20)

$1{,}000{,}000 \leq x_i \leq S_i + I_i - E_i - P_i - F_i$ .. (21)

$S_1 + I_1 + I_2 \geq 2{,}899{,}066 + S_3$ .. (22)

$S_1 + I_1 \geq 1{,}379{,}105$ .. (23)

$S_2 + I_2 \geq 1{,}519{,}961$ .. (24)

$S_3 \geq 1{,}515{,}315$ .. (25)

$2322 \leq F_i \leq 4644$ .. (26)

$376{,}339 \leq E_i \leq 514{,}871$ .. (27)

$444 \leq P_i$ .. (28)

$W_i \geq 0$ .. (29)

Algorithm for Solution

The objective function as depicted by Equation 17 is to be maximized by varying X_i. Equation 18 is the transformation equation which defines the state in which the reservoir will be at T_{i+1} if the reservoir state at T_i is S_i, X_i cubic metres of water are released, I_i cubic metres of water received as inflow, F_i cubic metres of sediments deposited, E_i cubic metres of water lost to the atmosphere, P_i cubic metres of water seeped into the ground and w_i cubic metres of water lost as flood overflow over the spillway. Equation 19 and 20 limit the state of the reservoir at times T_i and T_{i+1} respectively, to somewhere between empty and full. Equation 21 specifies the lower and upper limits on what the releases can be. Equation 22 states the minimum quantity of water the storage reservoir must handle for the entire planning period. Equations 23 and 24 specify the minimum quantity of water the storage reservoir must handle during the raining and dry seasons respectively.

Equation 25 specifies the minimum quantity of water that must remain in the storage reservoir at the end of the planning period. Equations 26 and 27, specify the lower and upper limits on the rate of siltation and evaporation respectively. Equation 28 specifies the lower limit on the rate of seepage, and equation 29 states that flood overflow cannot be negative.

RESULTS

Using the numerical values defined as input data in Tables 1 and 2, and equations 17 to 29, the dynamic programming problem was simulated using a Microsoft Watfor77 compiler on a Pentium III desktop computer. The computer program listings coded in FORTRAN language is in Appendix 1. The output of the simulation exercise as presented in Appendix 2 is rearranged in Table 3. For each combination of S_1, I_1 and I_2, the returns for the entire planning period are tabulated under appropriate sub-headings. A graphical concept with the optimal paths clearly shown has been used to amplify the results generated by the computer, as depicted by Fig. 1.

Table 3: Optimal Policies and Optimal set of States for the Various Combinations of S_1, I_1, and I_2.

S/N	Possible combination of initial reservoir state and hydrological inputs (m^3)			Optimal set of reservoir states (m^3)			Optimal series of water releases in m^3 (optimal policy)		Cumulative return (MU) from sales of water.
	S_1	I_1	I_2	S_1	S_2	S_3	X_1	X_2	
1	1398515	2097772	1398515	1398515	2097772	1515315	1019410	1461010	2267581
2	2097772	1398515	1398515	2097772	2097772	1515315	1019410	1461010	2267581
3	2097772	2097772	699257	2097772	2797029	1515315	1515315	1461010	2267581
4	2097772	2097772	1398515	2097772	2097772	1515315	1718667	1461010	5568075
5	2097772	2097772	1398515	2097772	2797029	1515315	1515315	1019410	5568075
6	2097772	2097772	1398515	2097772	2797029	2097772	1019410	1577811	2818880
7	2797029	699257	1398515	2797029	2097772	1515315	1019410	1577811	2818880
8	2797029	1398515	699257	2797029	2797029	1515315	1019410	1461010	2267581
9	2797029	1398515	1398515	2797029	2097772	1515315	1718667	1461010	5568075
10	2797029	1398515	1398515	2797029	2797029	1515315	1019410	2160268	5568075
11	2797029	1398515	1398515	2797029	2797029	2097772	1019410	1577811	2818880
12	2797029	2097772	699257	2797029	2797029	1515315	1718667	1461010	5568075
13	2797029	2097772	2097772	1398515	2797029	2097772	1515315	2417924	8868570
14	2797029	2097772	2097772	2797029	2797029	1515315	1718667	2160268	8868570
15	2797029	2097772	2097772	2797029	2797029	2097772	1718667	1577811	6119374

DISCUSSION

It can be observed from Table 3 that the minimum reservoir level at the end of the planning period has been maintained at 1,515,315m^3 of water to ensure that the university community has adequate supply of water in case of little flows for the next planning stage. In this regard, the optimal policy for the entire planning period is that path that led to the maximum cumulative return when the quantity of water in the reservoir is 1,515,315 m^3. A study of table 3, also reveals that the initial state of the reservoir at the beginning of the planning period as well as the probabilistic inflow due to the environmental input have had a strong influence on the optimal policy. These have been verified in terms of the number of possible combinations of S_1, I_1 and I_2 truncated by the computer simulation. While the inflow due to the environmental input can only be estimated via hydrological analysis of sufficient number of years of stream flow data, the scheduled release of water could be determined, guided by a desire to optimize returns. The series of releases made to fulfill this desire constitute the optimal policy, as depicted in table 3. All the computed values for the optimal set of reservoir states and optimal series of water releases have been confirmed to satisfy the constraints imposed on the objective function. It follows therefore, that for any given values of S1, I1, and I2, there is a corresponding optimal policy that will generate the maximum cumulative returns from sales of water.

The maximum cumulative return ever, which could be obtained, is 8,868,570MU, and this would occur when the reservoir is full at the beginning of the raining season, as illustrated in Fig. 1.

CONCLUSION

In this paper, dynamic programming technique has been used to optimize the reservoir hold-up and water releases throughout an operational twelve-month period. The optimal policy so derived could guide the management of Opa Waterworks on the quantity of water to release to optimize return from sales of water without jeopardizing the reservoir operations. The model has three major limitations. The first is that the model uses seasonal time steps. This precludes detailed consideration of hourly phenomena. Therefore, transient phenomenon such as flood peaks is simulated in an approximate way to include their overall impact on the seasonal results. The second is the deterministic character of the model which gives it complete foresight over the inputs. It allows the use of only one specified project year and one hydrologic year at a time. The effects of these limitations on the results have not been fully explored as only quarterly incremental reservoir capacities have been used. The third limitation is that a global optimum cannot be guaranteed as the quantity of inflow into the impounding reservoir for the planning period is uncertain. In the near future, when adequate and sufficient data have been gathered, it is recommended that shorter time steps and superior hydrologic input forecasts be used in the optimization process to improve the optimal policy.

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