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NON-COOPERATIVE GAME-BASED PARTNER SELECTION SCHEME IN USER COOPERATION NETWORKS

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ABSTRACT

The performance of any cooperative wireless communication system depends largely on the selection of a proper partner(s) by the source node to help it in forwarding information to the destination node. In this paper, we consider the concepts of partner (or relay node) selection and power allocation for a distributed communication network. A type of non-cooperative game referred to as Trade-Off game is employed so as to jointly consider the utilities of the source and relay nodes, where in this case, the source is the node that requires help with forwarding of its information while the partner is the node that is willing to help in forwarding the source node's information, but at a price. The approach enables the source node to maximize its utility by selecting a partner node based on (i) the proximity of the partner node to the source and destination nodes, and (ii) the price the partner node will charge for the help being rendered. Our proposed scheme helps the source locate and select the relay nodes at 'better' locations and purchase power optimally from them. It also aids the contending relay nodes in maximizing their own utilities as well by asking proper prices. Our game scheme is seen to converge to the unique equilibrium.

Key words: *cooperative communication, game theory, node, trade-off, utility.*

1. Introduction

Cooperative communications have recently gained prominence and much attention as an emerging strategy for transmission for next generation or future wireless networks. The basic idea behind this concept is that partner or relay nodes can act as virtual antenna arrays in helping the source node forward its information or data to the destination node. Through this, cooperative communication or cooperative diversity takes full advantage of the broadcast nature of wireless networks. It also exploits the spatial and multiuser diversity inherent in the traditional MIMO techniques, without each node necessarily having multiple antennas [Wang et al., 2009], [Elfituri et al., 2009].

The performance of cooperative communication largely depends on proper allocation of resources such as power and bandwidth, careful placement and selection of partners or relays. There are many protocols that have been devised for implementing cooperative diversity in wireless communications, some of which include the Amplify-and-Forward scheme, the Decode-and-Forward scheme, Estimate-and-Forward scheme and Coded cooperation. No matter the type of scheme or protocol employed in implementing cooperation, one thing that is sacrosanct is that the objective is to obtain a higher transmit diversity.

Recently, several works have dealt with the issue of partner selection and resource allocation in cooperative communications. These works are found to be in two categories namely, centralized (for example, [Ng and Wei, 2007; Yuan et al., 2010; Wenbing et al, 2010]) and decentralized (e.g.[Wang et al., 2009; Bletsas et al., 2006; Savazzi and Spagnolini, 2007; Lingjie et al., 2011; Yang et al., 2011; Niyato et al., 2009; Shaolei and Van der Schaar, 2011; Li et al., 2010]). There have been more researches on the distributed systems because they are more favorable in practical terms since they require only the local information of the nodes, unlike the centralized systems which require the global channel state information, and thus incur higher signaling overhead [Yuan et al., 2011]. For instance, in [Savazzi and Spagnolini, 2007], the authors proposed a partner selection scheme for distributed systems based on limited instantaneous SNR. The authors in [Bletsas et al., 2006] proposed a distributed power control framework for a single-source, multiple-relay system to optimize multihop diversity.

In the last few years, game theory has grown to be a veritable tool in the analysis of distributed systems due to their autonomous and self-configuring capability. For instance, in [[Wang et al., 2009] a non-cooperative game known as Stackelberg game was employed to develop a power allocation algorithm. The network is modeled as a single user, multi-relay system in which the source acts as the buyer node and the relays act as the sellers of resource (i.e. power). However, the buying and selling between the source and relay nodes activities in [[Wang et al., 2009] do not consider the proximity of the relay nodes to either the source or destination nodes. Thus, this incurs relatively longer convergence time and by extension higher energy consumption on the wireless network.

In this work we intend to improve on the partner selection scheme in [Wang et al., 2009] by considering proximity of the relay nodes to either the source or destination nodes prior to the start of the game, with a view to reducing the number of relay nodes available for the game. The new scheme developed is referred to as the Trade-Off game scheme.

In a nutshell, the major contributions of this research include reducing the convergence time in the partner selection process by the source node with a view to having a prudent utilization of power on the network. This involves the introduction of proximity criteria for the partner selection as well as proposing a power allocation scheme and an analytical derivation for the equilibrium.

The rest of this paper is organized as follows: Section 2 presents the background to this work. The proposed partner selection scheme is described in Section 3 while Section 4 gives the proposed power allocation scheme. Section 5 describes the equilibrium of our proposed game scheme. In Section 6 we have the results and discussion. The conclusion is given in Section 7.

2. Background

2.1 Cooperative Communication System Model

A time division model of cooperative communication scheme is as shown in Fig. 1a while a multi-node cooperative model is shown in Fig. 1b, with multiple relay nodes. The cooperative process is carried out over two-frame transmissions as shown in Fig. 1a. For the purpose of improvement of the overall performance through diversity, the cooperation is done by sending data from the source node to the destination node in the first frame or time slot while the data is sent via the relay or partner node to the destination node in the second time slot.

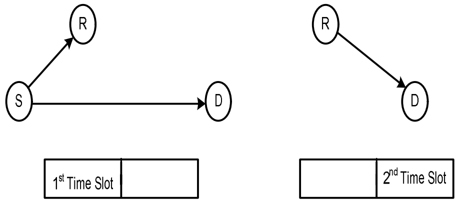

Fig. 1a A 3-node cooperative system model in the time division mode

2.2 Stackelberg Game Model

In non-cooperative games, there is the possibility of existence of hierarchy among the players in the game whereby one or more players declare and announce their own strategies prior to the other players announcing theirs. Put in another way, these other players respond or react to the strategies declared by the former players. In a hierarchical situation such as this, the declaring players can be in a position to enforce their own strategies upon the other players.

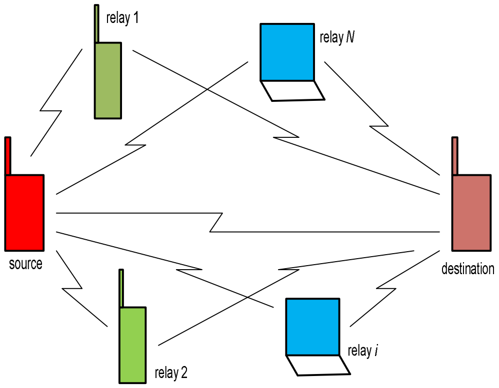

Fig.1b Multi – relay node cooperative communication scheme

As such, the player who holds this strong position which can be imposed on others is called the leader while the other players who react or respond to this leader's declared strategy are called followers. Thus a Stackelberg game is a non-cooperative leader-follower(s) game. However in some cases, there could be multiple leaders and followers [Han et al., 2012], [Hua and Junhu, 2008].

Given two players in a non-cooperative game which involves a leader and follower, whose strategies are respectively denoted by S_1 and S_2, whenever the leader with strategy S_1 declares to play a particular strategy $s_1 \in S_1$, the player must also react or respond accordingly with another given strategy $s_2 \in S_2$. It is also possible that the follower may have many possible reactions to a given strategy of the leader. In view of this, the following definitions are given for the Stackelberg strategy, according to Han et al., 2012]

Def.1: Given a finite 2-person game, the set $R_2(s_1)$, defined for each strategy $s_1 \in S_1$ by

$$R_2(s_1) = \{s_2 \in S_2 : u_2(s_1,s_2) \geq u_2(s_1,t), \forall t \in S_2\} \qquad (1)$$

is the optimal response (or reaction) set of player 2 to the strategy $s_1 \in S_1$ of player 1

Def.2: In a finite game of two players, with player 1 as the leader and player 2 the follower, a strategy $s_1^* \in S_1$ is called a Stackelberg strategy (or Stackelberg equilibrium strategy) for the leader, if

$$\min_{s_2 \in R_2(s_1^*)} u_1(s_1^*, s_2) = \max_{s_1 \in S_1} \min_{s_2 \in R_2(s_1)} u_1(s_1, s_2) \underset{=}{\Delta} u_1^* \qquad (2)$$

In definition 2 above, the quantity u_1^* is the Stackelberg utility for the leader; this definition also applies if the player 2 is the leader and 1 the follower. However, this Stackelberg strategy proves to be a useful tool in defining equilibrium points in games that are hierarchical in their decision-making.

However, in a multiuser cooperative communication networks, modeled as a network consisting of three nodes, namely, source, relay and destination nodes, the Stackelberg game is usually employed in order to jointly consider the benefits of the source and relay nodes in cooperative communication [Wang et al., 2006]. In this case, the game is referred to as buyer-seller game instead of the former leader-follower game. Actually the buyer is the leader while the seller is the follower. This is so because it is the leader that broadcasts the desire to buy either power or bandwidth, or that requires the service of one or more of the relays to help it forward its data onward to the destination. Thus the source is the buyer while the relay(s) is (are) the seller(s) in the game.

The Stackelberg game is divided into two levels, which are the *source node-level* game and the *partner node-level* game. In this game, it is noteworthy that each of the players involved is selfish and wishes to maximize its own benefit independent of the other players, and this is what is referred to as contention or tension among the players. Just as in normal economic concepts, the buyer (source node) aims to get most benefits at the least payment, while each relay aims at earning the payment put forward by the source, which not only covers their cost for the help or service rendered, but also gain as much extra profit as possible [Wang et al., 2009; Wang et al., 2006]

2.3 Achievable Transmission Rate Capacity

A simple cooperative model as depicted in Figure 1a where there is one relay node and one source node in time division mode is considered and is shown in Appendix A. The schematic in Figure 1b shows a single source node and *N*-relay nodes. The cooperative protocol used here is the Estimate and Forward (EF) protocol. This protocol is used unlike in previous work where the Amplify and Forward scheme is used, so as to bypass the errors and noise which are usually amplified with the original transmitted signal.

In the first time slot or Phase 1 (in Figure 1a), the source node (s) broadcasts its information, and is received by the both the partner (r) and destination (d) nodes as described in Appendix A. The achievable transmission rate C_T at the destination node is the rate for the source node (s) – relay node (r) – destination node (d) channels while the C_{DT} is the rate for the direct transmission (DT) or source node (s) – destination node (d) channels.

For the direct transmission, the achievable transmission rate or channel capacity is given as

$$C_{DT} = W \log_2\left(1 + \gamma_{DT}\right) \qquad (3)$$

where W is the bandwidth of the transmitted signal from the source node and γ_{DT} is the SNR for the source – destination channel.

For the source node-relay node-destination node channel, the following achievable transmission rate capacity C_T is applicable

$$C_T = W \log_2\left(1 + \gamma_{DT} + \gamma_T\right) \qquad (4)$$

where γ_T is the total SNR for the source node-relay node-destination node channels

3. Proposed Partner Selection Scheme

In a cooperative communication set-up consisting of the source, partner (relay) and destination nodes, the relay nodes are randomly distributed at different points on the network layout; from where help is required of them by the source to help it forward data or information to the destination terminal, and at the same time, the relay nodes ask different prices for helping the source node forward its data, using the economic game concept of buying and selling and trade-off. However, since these relay nodes are randomly distributed, there is the tendency that some of them would be closer to either the source node or destination node than others, and this proximity will be an important factor in the price being asked by the relay nodes. So, in the light of this, our proposed partner selection scheme will be based on three criteria unlike [Wang et al., 2009] which is based on the concept of buying and selling. These are as follows:

1. The proximity of the relay node to the source node.
2. The proximity of the relay node to the destination node.
3. The price being asked by the relay node for the source node to pay for forwarding its information to the destination node.

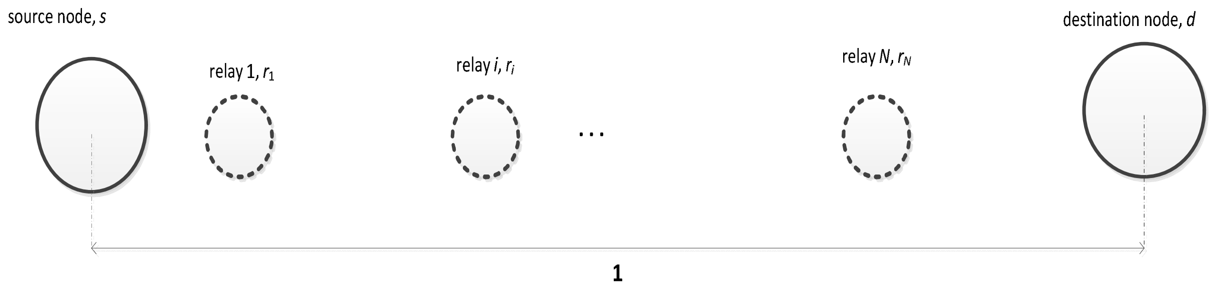

Fig. 2 Illustration of the relative distances of the relay nodes from the source and destination nodes

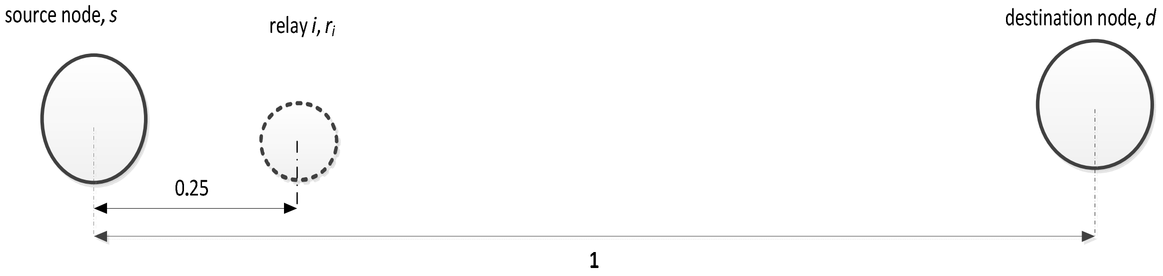

Fig. 3 Illustration of the proximity of a relay node to the source node

3.1 The Proximity of the relay node, r_i to the source node, s

We assume the source node, *s* and the destination node, *d* are separated by a normalized distance of 1. We also assume that the relay nodes are located at different points along the path of distance *x* for the purpose of this analysis. It is also assumed that there are *N* relay nodes available for this selection game. This is illustrated in Fig. 2. As can be seen from the figure, if a relay node r_i is situated at a position very close to the source node and by effect very far from the destination node, the relay node may be compelled to ask a low price from the source node so as to make it buy power from it. It would now depend on the source node to either buy power at this low price or not and increase its utility at the expense of the distance the signal will travel before reaching the destination, bringing up the concept of trade-off into the picture. It would also mean that as that relay node moves away from this location, the source node may have no more incentive to buy power from it.

If we assume that relay nodes midway between the source and destination nodes are at a point 0.25 from both ends, then we can comfortably say that a relay node very close to the source node is at a distance $0 < x < 0.25$ from the source node end. Therefore, if the source node would select any relay node located very close to it, to enjoy the low price it would offer due to how far it is from the destination, then it would select the relay nodes located at a distance < 0.25 from the source node. The flow chart for this criterion is given in Fig. 4.

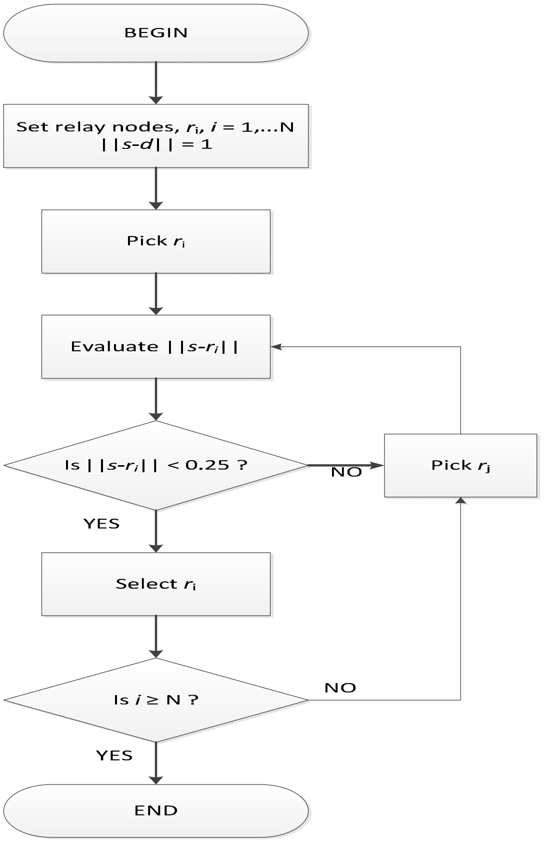

Fig.4 Flow chart for the criteria of proximity of the relay node, r_i to the source node, *s*

3.2 The Proximity of the relay node, r_i to the destination node, d.

Assumptions are also made as was done in 3.1, but now focus is on the relay nodes being closer to the destination node, *d* rather than being close to the source node, *s*. This would mean that any node whose distance from the source node is greater than 0.75 would be considered very close to the destination node; and so the source node has the option of ether selecting these relay nodes or not. As mentioned earlier, the contention for the eventually selected node will be between the relay nodes at these extreme ends: very close to the source node and very close to the destination node. In explaining what will likely happen at this location, it can be inferred from Fig. 5 that the relay node would need a little amount of power to forward the source node's data to the destination node, and as such, for it to maximize its own utility or benefit, it will ask for a high price, *p* from the source node. It would now be left for the source node to decide whether to purchase this power at such a price or not.

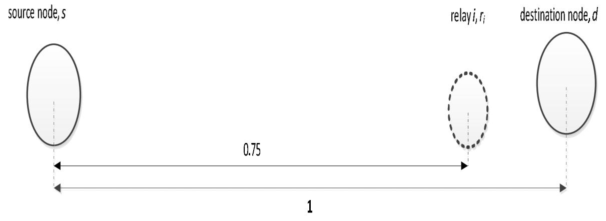

Fig. 5 Illustration of the proximity of a relay node to the destination node

The flow chart of what happens in this scenario is given in Fig. 6. It is noteworthy that *L* is the number of relay nodes left after the source node has selected the relay nodes close to the source node, *s* in the first criterion.

3.3 The Issue of Price

The issue of price in our proposed scheme now comes up after the first two criteria have been validated. From the relay nodes selected from the first and second criteria, we now constitute a set L_s which contains all these relay nodes, from which the final 'best' relay node would now be selected for cooperation. At this point, the contention or tension would now be among these relay nodes who are contending for the final selection by the source node.

Apart from the fact that the source node seeks the 'best partner(s)' with which to cooperate with, it also tries to maximize its utility or payoff, U_s obtainable in the game. The source node achieves this by purchasing a maximum amount of power from the selected relay node – this power bought by the source is increased gradually until a maximum is reached.

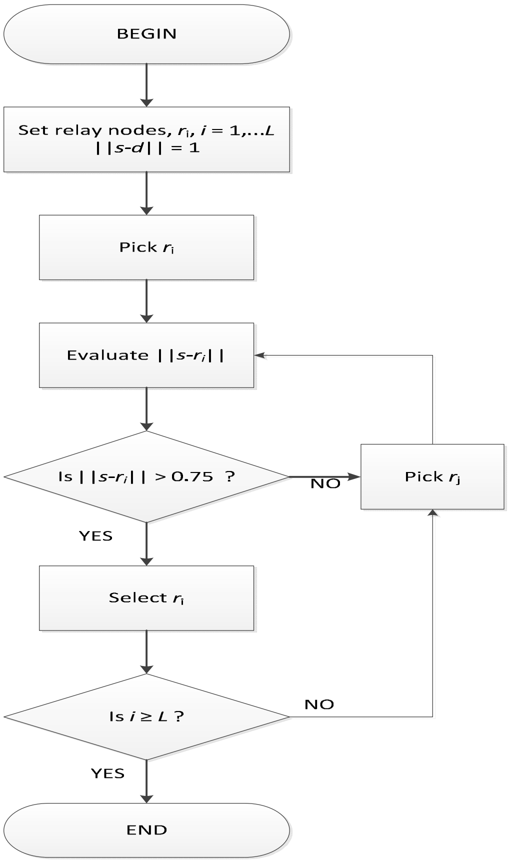

Fig. 6 Flow chart for the criteria of proximity of the relay node, r_i to the destination node, *d*.

Two parameters are needed in the analysis of this criterion of price. These are i.) the transmission rate capacity for a communication network, described in II.C and ii) the variation of the utility of the source node with the power purchased from the relay nodes. From the definition of utility

$$U_s = g\Delta C_* - \psi \tag{5}$$

where

$$\Delta C_* = C_T - C_{DT}$$

and

$$\psi = \sum_{i=1}^{L_{sp}} p_i P_{r_i}$$

where g = gain per unit of rate and ψ = total payments made to the relay nodes by the source node.

Since $\psi = p_i P_{r_i} + p_i P_{r_i} + \ldots$, Eq. (5) can be rewritten as follows

$$U_s = g\Delta C_* - p_i P_{r_i} \tag{6}$$

To obtain how the utility of the source node *s* varies with the purchased power from the relay nodes, the following derivatives are obtained:

$$\frac{\partial U_s}{\partial P_{r_i}} = g\frac{\partial \Delta C_*}{\partial P_{r_i}} - p_i \frac{\partial P_{r_i}}{\partial P_{r_i}} \qquad (7)$$

which yields

$$\frac{\partial U_s}{\partial P_{r_i}} = g\frac{\partial \Delta C_*}{\partial P_{r_i}} - p_i \qquad i = 1, 2, \dots L_{sp}$$

This assumes there are L_{sp} relay nodes available for the selection game at this time which is after the first two criteria. The ΔC_* is as defined earlier, g refers to the gain per unit of rate, and ψ stands for the total payments made by the source node s to the relay nodes to buy power, as defined earlier where p_i denotes the price per unit of power being sold by relay node r_i to source node s, and P_{r_i} refers to the amount of power source node s is buying from relay node r_i.

Beginning at $P_{r_i} = 0$, if $p_i < g\frac{\partial \Delta C_*}{\partial P_{r_i}}$ for a particular relay node r_i, it is clear that $\frac{\partial U_s}{\partial P_{r_i}} > 0$ (for it would mean that $\frac{\partial U_s}{\partial P_{r_i}} = +ve$) which also means that a higher utility U_s will be obtained by the source node when a higher amount of power, P_r is bought; else, that relay node r_i is exempted or excluded from participating in the game at that time.

The relay node selection strategy based on the utility obtainable by the source node as a result of the price announced by the relay node and the power the source node is able to buy from it is shown in the flow chart of Fig. 7.

3 Power Allocation Scheme

In a cooperative communication set-up consisting of the source, partner (relay) and destination nodes, the relay nodes are randomly distributed at different points on the network layout; from where help is required of them by the source to help it forward data or information to the destination terminal, and at the same time, the relay nodes ask different prices for helping the source node forward its data, using the economic game concept of buying and selling and trade-off. However, after the suitable partners have been selected, of necessity afterwards is the allocation of power to these selected relay nodes.

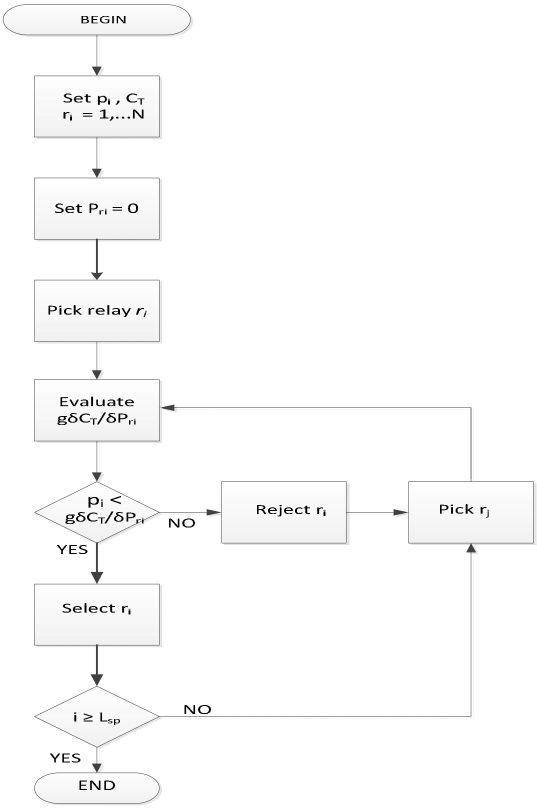

Fig. 7 Flow chart for selecting a suitable partner node based on price and power purchased

In this subsection that follows we describe two levels of this game scheme, preparatory to the development of the power allocation scheme. These are the source node-level and relay node-level games.

3.1 Source node as Buyer of Resource

Modeling the source node as buyer of resource, for example, power is aimed at making the buyer obtain the most benefits at least possible payments, similar to what happens in normal business concept of buying and selling. The source, s has its utility function defined as [Wang et al., 2009]

$$U_s = gC_T - \psi \qquad (8)$$

where, as mentioned previously, C_T denotes the transmission rate capacity achievable at the MRC output, with the help of the relaying partners, g refers to the gain per unit of rate, and ψ stands for the total payments paid by the source s to the relay nodes to buy power, given by

$$\psi = \sum_{i=1}^{N} p_i P_{r_i} \qquad (9)$$

where p_i denotes the price per unit of power being sold by relay r_i to source *s*, and P_{r_i} refers to the amount of power node *s* is buying from relay r_i. And because the source node will want to maximize its resource, an optimization problem is formulated thus:

$$\max U_s = gC_T - \psi \quad \text{s.t.} \quad P_{r_i} \geq 0 \tag{10}$$

For a single-relay case, the optimization problem is given as

$$\max U_s = gC_T - p_1 P_{r_1}, \quad \text{s.t.} \quad P_{r_1} \geq 0 \tag{11}$$

then

$$\max U_s = gC_T - (p_1 P_{r_1} + p_2 P_{r_2}), \quad \text{s.t.} \quad P_{r_1} \geq 0 \tag{12}$$

For a two-relay node case, and for *n*-relay case, it becomes

$$\max U_s = gC_T - \sum_{i=1}^{n} p_i P_{r_i}, \quad \text{s.t} \quad P_{r_i} \geq 0 \tag{13}$$

4.2 Relay Node as Seller of Resource

Every relay node in the cooperative process is seen as a seller of resources, who seeks to sell its resources, for example, power in this case, and also targets, not only receiving the payment for the cost of forwarding data to the destination node for the source node, but also earning much profit from the deal.

Then just as in the buyer's case, the utility of the relay node r_i will be given as [1]

$$U_{r_i} = p_i P_{r_i} - a_i P_{r_i} \tag{14}$$

where a_i is a parameter denoting the cost of the power for forwarding data by the relay *i*. Also, since the relay will also try to maximize its opportunity, the optimization problem is written thus

$$\max U_{r_i} = (p_i - a_i) P_{r_i}, \quad \text{s.t.} \quad p_i > 0, \quad \forall i \tag{15}$$

From the above discussions, it can be seen that the two games, the buyer-level and seller-level games are aimed at (i) selection of partners by the source node (ii) deciding the optimal price p_i to maximize the partners' (relays') profits or utility, U_{r_i}; and (iii) getting the corresponding optimal power that will be consumed to maximize its (source's) utility U_s. It is also noteworthy that the price p_i and knowledge of the amount of power P_{r_i} the source would buy from the relay node are the two signallings required for data exchange between the source and relay nodes.

Actually, apart from the fact that the source node seeks the 'best partner(s)' with which to cooperate with, it also tries to maximize its utility or payoff, U_s obtainable in the game. The source node achieves this by purchasing a maximum amount of power from the selected relay node – this power bought by the source is increased gradually until a maximum is reached.

After selecting all the 'suitable' partners for the cooperation, what is next is how to allocate resource to these selected relay nodes. This is important in that in any power-limited network, there is the need to ascertain the optimal power that can be allocated, or in our work, the optimal power that can be sold or bought by either the relay node or source node respectively. This is the major focus of this work.

In the event that the suitable partner(s) have been selected by the source node based on the criteria discussed in Section III and a selected node set constituted as

$$L_{sp} = \{r_1, r_2, r_3, \ldots r_{L_s}\}$$

where L_{sp} denotes the number of selected partner nodes (based on the second criteria for selection discussed earlier), there is the need to compute the optimal value of the resource (in this case, power) that the partner can offer the source node to enable it maximize its utility. This is known as the optimum resource or simply optimum power allocation, in case of power as the metric; so, in this work, an optimum power allocation scheme is developed for the cooperative scheme. The execution of this scheme is however preceded by the partner selection scheme developed in the previous section.

Recall that during the partner selection scheme, it is the variation of the source node's utility U_s to the power *P* of the partner node that gives rise to the criteria for the selection of suitable partner(s) for the source node after the first two selection criteria based on proximity to the source and destination nodes have been considered. That is

$$\frac{\partial U_s}{\partial P_{r_i}} = g \frac{\partial C_T}{\partial P_{r_i}} - p_i = 0 \tag{16}$$

where *g* is the gain per unit of rate at the output of the MRC (receiving end) and p_i is the price per unit of power bought by the source node

$$C_T = W \log_2\left(1 + \left(\gamma_{sd} + \gamma_{rd}\right)\right)$$

$$C_T = W \log_2\left(1 + \gamma_{sd} + \gamma_{rd}\right)$$

for a one-relay system; and

$$C_T = W \log_2\left(1 + \gamma_{sd} + \sum_{r_i \in L_s} \gamma_{rd}\right) \tag{17}$$

for L_{sp} selected relay nodes

Thus,

$$gC_T = gW \log_2\left(1 + \gamma_{sd} + \sum_{r_i \in L_s} \gamma_{r_i d}\right) \tag{18}$$

For computational simplicity, we assume

$$W' = \frac{gW}{In2} \text{ ; and } 1 + \gamma_{sd} = D$$

We can write

$$gC_T = gW \log 2\left(D + \sum_{r_i=1}^{L_s} \gamma_{r_i d}\right)$$

(19)

Then

$$gC_T = W' In\left(D + \sum_{r_i=1}^{L_s} \gamma_{r_i d}\right) \tag{20}$$

Finally

$$gC_T = W' InD + W' In\left(1 + \gamma'_{tot}\right)$$

(21)

where γ'_{tot} is the total SNR for all the partner-destination channels in the cooperative network (i.e.)

$$\gamma'_{tot} = \sum_{r_i=1}^{L_s} \gamma'_{r_i d} = \frac{1}{D} \sum_{r_i=1}^{L_s} \gamma_{r_i d} \tag{22}$$

and $\gamma'_{r_i d}$ is written as

$$\gamma'_{r_i d} = \frac{\alpha_i}{1 + \dfrac{\beta_i}{P_{r_i}}} = \frac{\alpha_i \cdot P_{r_i}}{P_{r_i} + \beta_i}$$

(23)

where $\alpha_i = \dfrac{E_{sd} h_{sd}{}^2}{N_o + E_{sd} h_{sd}{}^2}$ and $\beta_i = \dfrac{E_{sd} h_{sr}{}^2 + N_o}{h_{rd}{}^2}$

Recall that

$$M = \sum_{r_i \in L_s} p_i P_{r_i} = p_1 P_{r_1} + p_2 P_{r_2} + \ldots p_{N_s} P_{r_{N_s}} \tag{24}$$

Substituting Eq. (24) and Eq. (20) into Eq. (16),

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{W'}{\left(1 + \sum_{k=1}^{N_s} \dfrac{\alpha_k P_{r_k}}{P_{r_k} + \beta_k}\right)} \frac{\alpha_i \beta_i}{\left(P_{r_i} + \beta_i\right)^2} - p_i = 0 \tag{25}$$

Rearranging Eq. (25), we have

$$\frac{p_i}{\alpha_i \beta_i}\left(P_{r_i} + \beta_i\right)^2 = \frac{W'}{\left(1 + \sum_{k=1}^{N_s} \dfrac{\alpha_k P_{r_k}}{P_{r_k} + \beta_k}\right)} \tag{26}$$

From Eq. (26), it is seen that for any partner *i* on the LHS, the RHS is the same, so

$$\frac{p_i}{\alpha_i \beta_i}\left(P_{r_i} + \beta_i\right)^2 = \frac{p_j}{\alpha_j \beta_j}\left(P_{r_j} + \beta_j\right)^2 \tag{27}$$

where *i*, *j* represent two different partners in the selected partners set. Solving Eq. (12),

$$p_i \alpha_j \beta_j \left(P_{r_i} + \beta_i\right)^2 = p_j \alpha_i \beta_i \left(P_{r_j} + \beta_j\right)^2$$

$$\frac{p_i \alpha_j \beta_j}{p_j \alpha_i \beta_i}\left(P_{r_i} + \beta_i\right)^2 = \left(P_{r_j} + \beta_j\right)^2$$

$$\sqrt{\frac{p_i \alpha_j \beta_j}{p_j \alpha_i \beta_i}}\left(P_{r_i} + \beta_i\right) = \left(P_{r_j} + \beta_j\right)$$

Thus

$$P_{r_j} = \sqrt{\frac{p_i \alpha_j \beta_j}{p_j \alpha_i \beta_i}}\left(P_{r_i} + \beta_i\right) - \beta_j \tag{28}$$

We then substitute Eq. (28) into Eq. (23) and simplify to obtain

$$\gamma'_{r_j d} = \left(\frac{\alpha_j}{1 + \dfrac{\beta_j}{P_{r_j}}}\right) = \left(\alpha_j - \sqrt{\frac{p_j \alpha_i \beta_i}{p_i \alpha_j \beta_j}} \frac{\alpha_j \beta_j}{\left(P_{r_i} + \beta_i\right)}\right) \tag{29}$$

Recalling also from Eq. (29) that $\gamma'_{tot} = \sum_{i=1}^{N_s} \gamma'_{r_i d}$, we have from Eq. (35),

$$\gamma'_{tot} = \begin{bmatrix} \left[\alpha_1 - \sqrt{\dfrac{p_1 \alpha_i \beta_i}{p_i \alpha_1 \beta_1}} \dfrac{\alpha_1 \beta_1}{\left(P_{r_i} + \beta_i\right)}\right] + \left[\alpha_2 - \sqrt{\dfrac{p_2 \alpha_i \beta_i}{p_i \alpha_2 \beta_2}} \dfrac{\alpha_2 \beta_2}{\left(P_{r_i} + \beta_i\right)}\right] + \\ \left[\alpha_1 - \sqrt{\dfrac{p_3 \alpha_i \beta_i}{p_i \alpha_3 \beta_3}} \dfrac{\alpha_3 \beta_3}{\left(P_{r_i} + \beta_i\right)}\right] + \ldots \\ \ldots + \left[\alpha_1 - \sqrt{\dfrac{p_{N_s} \alpha_i \beta_i}{p_i \alpha_{N_s} \beta_{N_s}}} \dfrac{\alpha_{N_s} \beta_{N_s}}{\left(P_{r_i} + \beta_i\right)}\right] \end{bmatrix} \tag{30}$$

Substituting Eq. (33) into Eq. (29), and after some manipulations, we have,

$$\left[\begin{array}{l}\left(1+\sum_{j=1}^{N_s}\alpha_j\right)\left[\sqrt{\frac{p_i}{\alpha_i\beta_i}}\left(P_{r_i}+\beta_i\right)\right]^2- \\ \sum_{j=1}^{N_s}\sqrt{p_j\alpha_j\beta_j}\left[\sqrt{\frac{p_i}{\alpha_i\beta_i}}\left(P_{r_i}+\beta_i\right)\right]-W'\end{array}\right]=0 \qquad (31)$$

which is a quadratic function in P_{r_i}

Therefore, for a selected partner, the optimum power allocation or consumption to enable the source node maximize its utility is given as follows:

$$P_{r_i}=\left[\frac{\sqrt{\frac{\alpha_i\beta_i}{p_i}}}{2\left(1+\sum_{j=1}^{N_s}\alpha_j\right)}\left(\sum_{j=1}^{N_s}\sqrt{p_j\alpha_j\beta_j}+\sqrt{\left(\sum_{j=1}^{N_s}p_j\alpha_j\beta_j\right)^2+4\left(1+\sum_{j=1}^{N_s}\alpha_j\right)W'}-\beta_i\right)\right] \qquad (32)$$

The solution in Eq. (32) is actually a general solution for the optimum power allocation for the selected partner, which is also seen to be the global optimum to the optimization problem in Eq. (16). However, the value may be negative if for instance the channel condition is poor or too high a price is asked by the partner node which might be unaffordable by the source node. So the optimum power allocation is modified as follows;

$$P_{r_i}^{opt} = \max(P_{r_i}, 0) \qquad (33)$$

where P_{r_i} is the solution of Eq. (38).

Now to the issue of the optimal price that the relay can ask: We substitute Eq. (32) into Eq. (11), giving us

$$\max U_{r_i} = (p_i - a)P_{r_i}^{opt}\left(p_1, \ldots p_2, \ldots p_i, \ldots p_{L_s}\right),\ p_i > 0, \forall i \qquad (34)$$

Being a game of tension among the relay nodes, trade-off thus exists between the utility U_{r_i} of the relay node and the price p_i as can be seen in Eq. (34). As discussed earlier, if a relay node is located very close to the destination node, there is every tendency for it to ask a high price from the source node so as to maximize its own utility; therefore, there should be an optimal price for the relay node to ask for. Aside that, this optimal power is also affected by the prices of the other relay nodes, since this is a game in which the source node only chooses the beneficial relay nodes as cooperating partners.

Thus, differentiating the relay node's utility U_{r_i} with respect to price p_i, we obtain

$$\frac{\partial U_{r_i}}{\partial p_i} = (p_i - a)\frac{\partial U_{r_i}}{\partial p_i} + P_{r_i}^{opt} \quad \forall r_i \quad i \in L_s \qquad (35)$$

Equating Eq. (35) to zero gives

$$(p_i - a)\frac{\partial U_{r_i}}{\partial p_i} + P_{r_i}^{opt} = 0 \qquad (36)$$

Solving Eq. (36) for p_i gives the optimal price a relay node can ask for, under the circumstance that the channel condition is good and that the relay node is in the proximity of the destination node. So this optimal price is denoted by

$$p_i^{opt} = p_i^{opt}(n, \{G_{sr}\}, \{G_{rd}\}) \qquad (37)$$

4 Equilibrium for the Proposed Scheme

An important factor to consider in order to validate the proposed game scheme is the equilibrium. There is the need to verify that the solution in Eq. (32) is the equilibrium which is termed Trade-Off Equilibrium (TE). First the definition of the TE for the proposed Trade-Off game is given as follows, using an idea from [Wang et al., 2009]:

(i) $P_{r_i}^{TE}$ is the TE of the proposed game (when p_i is fixed) if

$$U_s\left(\left\{P_{r_i}^{TE}\right\}\right) = \text{lub}\ U_s\left(\left\{P_{r_i}\right\}\right), \forall r_i \in L_s, \left\{P_{r_i}\right\} \geq 0 \qquad (38)$$

(ii) p_i^{TE} is the TE of the proposed game (P_{r_i} fixed) if

$$U_{r_i}\left(\left\{p_i^{TE}\right\}\right) = \text{lub}\ U_{r_i}\left(\left\{p_i\right\}\right), \forall r_i \in L_{sp}\ p_i > q_i \qquad (39)$$

where L_s and L_{sp} denote respectively the number of relay nodes available for the game and the relay nodes eventually selected by the source node while q_i denotes the least cost placed by relay node r_i at the commencement of the game and ***lub*** refers to *least upper bound* or supremum.

Then by differentiating the source node utility with respect to the relay node's power, i.e. $\frac{\partial U_s}{\partial P_{r_i}}$, and equating to zero, $P_{r_i}^{\max}$ as given in Eq. (32) can be solved. This is borne out of the fact that the utility function U_s, of the source node is jointly concave in $P_{r_i}, \forall i$, with $P_{r_i} \geq 0$ and with fixed p_i. This is shown as follows:

After taking the derivative of the source node's utility U_s with respect to the relay node's power, the 2nd-order derivative is then taken:

$$\frac{\partial^2 U_s}{\partial P_{r_i}{}^2} = \frac{W'}{\left(1+\sum_{k=1}^{N_s}\frac{\alpha_k P_{r_k}}{P_{r_k}+\beta_i}\right)^2}\left[\frac{\alpha_i\beta_i}{\left(P_{r_i}+\beta_i\right)^2}\right]^2 - 2\frac{W'}{\left(1+\sum_{k=1}^{N_s}\frac{\alpha_k P_{r_k}}{P_{r_k}+\beta_k}\right)}\frac{\alpha_i\beta_i}{\left(P_{r_i}+\beta_i\right)^3} \quad (40)$$

Also,

$$\frac{\partial^2 U_s}{\partial P_{r_i}\partial P_{r_j}} = -\frac{W'}{\left(1+\sum_{k=1}^{N_s}\frac{\alpha_k P_{r_k}}{\left(P_{r_k}+\beta_i\right)}\right)^2}\times\frac{\alpha_i\beta_i}{\left(P_{r_i}+\beta_i\right)^2}\frac{\alpha_j\beta_j}{\left(P_{r_j}+\beta_j\right)^2} \quad (41)$$

From earlier discussions and definitions, it could be seen that $W' > 0$, $\alpha_i > 0$, $\beta_i > 0$, $P_{r_i} \geq 0$. Because of this, $\frac{\partial^2 U_s}{\partial P_{r_i}^2} < 0$ as well as $\frac{\partial^2 U_s}{\partial P_{r_i}\partial P_{r_j}} < 0$. It is therefore without ambiguity to verify that

$$\frac{\partial^2 U_s}{\partial P_{r_i}^2}\frac{\partial^2 U_s}{\partial P_{r_j}^2} - \frac{\partial^2 U_s}{\partial P_{r_i}\partial P_{r_j}} > 0, \forall i, \forall j, i \neq j \quad (42)$$

It can also be seen that there is continuity between the source node's utility, U_s and the relay node's power, P_{r_i}, or in other words, U_s is continuous in P_{r_i}; and as such, if $P_{r_i} \geq 0$, it means U_s is strictly concave in P_{r_i}, $\forall i$.

From the foregoing, $P_{r_i}^{\max}$ in Eq. (32) can be said to be the global maximum that enables the maximization of the source node's utility, U_s via the allocation of power to the selected relay node(s). Therefore $P_{r_i}^{\max}$ is the trade-off equilibrium (TE) since it satisfies Eq. (38).

On the other hand, talking of practically implementing the proposed game scheme, the source node can as well obtain the maximal amount of power from the relay node(s) by increasing the amount bought gradually until the source node utility U_s attains its maximum.

5 Results and Discussions

Fig. 8 shows the interaction between the unit price of power and the utility of the source node. At a low price, the source node is willing to buy more power, thus enhancing its utility while also boosting the utility of the relay node as well, because it is the seller of the power. But as it can be seen from the plots (Fig. 8), as the price announced by the relay nodes begins to increase, the utility enjoyed by the source by being able to buy power also begins to reduce. Moreover, as it is seen that the source tends to derive more utility from relay node 1 than from relay node 2, the source would select relay node 1 as its cooperative partner.

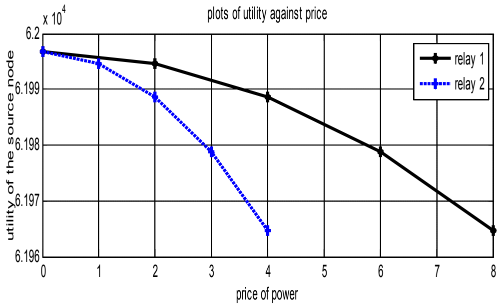

Fig.8. Plots showing the variation of the source node utility with the unit price of power

In Fig. 9, we show that as the relay node moves closer to the destination node, the utility of the relay node increases. This is so because of the fact that a little amount of power is needed by the relay to forward the source node's data to the destination; and as a result of this, the relay node would want to exploit the situation by asking a high price from the source node.

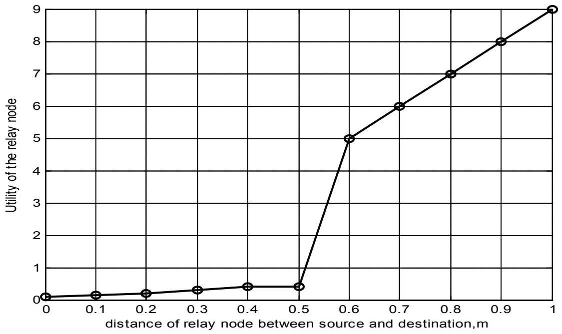

Fig. 9 Plot showing the variation of the relay node utility with the relay node at different locations between the source and destination nodes. Source node is located at (0,0) while destination node is at (1,0)

In Fig. 10, the optimum price that may be asked by the relay node is shown. As mentioned earlier, it is assumed that the relay node moves along a line [-2.0, 2.5]. As the plot shows, when the relay node is close to the source node (1,0), it can, with much efficiency, help forward source node's data; so, due to that location, the relay node will ask a low price so as to attract the source node in buying more resource. However when the location of the relay node is now close to the destination node at (0, 0), very small amount of power will be needed to forward source node's data. As a result, the relay node will try to exploit the situation by asking a high price so as to gain more incentive through the selling of the small amount of resource.

Moreover when the relay node moves further away from the destination node, the source node will cease from buying resource because that present location of the relay node will not make it any beneficial for the source node to buy resource.

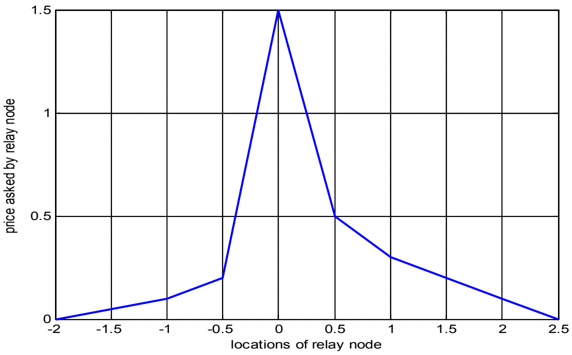

Fig. 10 Plot showing the optimal price and relay node at different locations

Fig. 11 gives the plots of the source node's utility against the iteration index, shows how at different prices, the source node's utility converges to the equilibrium. It was also observed that the time of convergence in our proposed scheme is lower than in a related work [1]. This is due to the introduction of the proximity criteria into our scheme, which rejected the not-so-beneficial relay nodes from participating in the game.

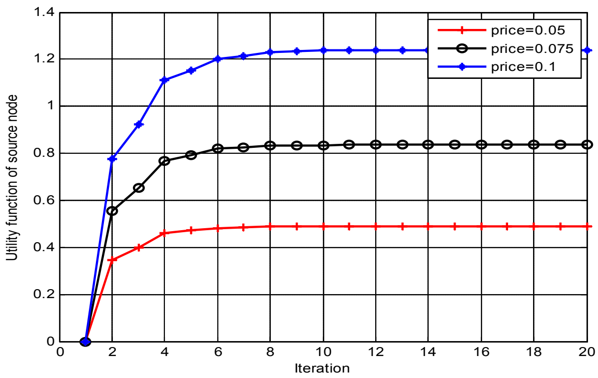

Fig. 11 Plots showing the convergence of the source node's utility to the Nash equilibrium

Fig. 12 shows the plots comparing the convergence speeds of our proposed game scheme with that in the work of Wang et al [1]. Our results show that the convergence is faster in our scheme primarily due to the introduction of the proximity criteria. For example, for the price p = 0.1, our scheme converges after 5 iterations against the 7 iterations in the other work. The plots also show that there is a higher utility for the source node at a lower price.

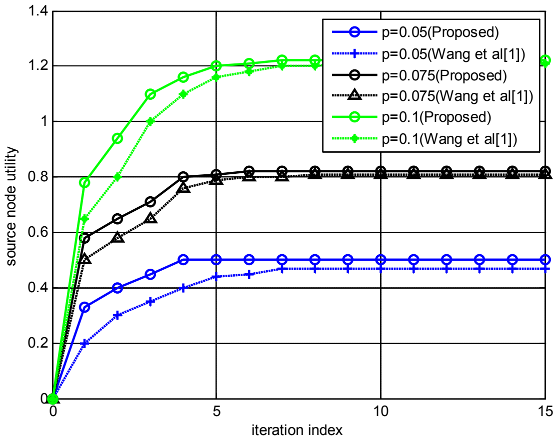

Fig. 12 Plots comparing the convergence speeds of the proposed game scheme with the scheme in Wang et al [1]

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