LAUTECH Journal of Engineering and Technology 17 (2) 2023: 104-110

PARAMETER ESTIMATION OF ARIMA USING GOAL PROGRAMMING

[1]*Efuwape B. T., [1]Adeniran S. K., [1]Ogunwobi, Z. O., [1]Abdullah, K-K. A., [1]Olasupo, A. O[1]., and [2]Efuwape T. O.

[1,3]Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, Nigeria.
[2]Academic Planning Unit, Bells University of Technology, Ota, Nigeria.
**Corresponding author: efuwape.biodun@oouagoiwoye.edu.ng*

ABSTRACT

Goal programming (GP) is an improved technique of the linear programming model that is suitable for multi-criteria decision making for organizations that have multiple objectives that are usually not measurable in the same units. Autoregressive integrated moving average model (ARIMA) is useful in predicting future behavior based on past behaviors. It is also useful for forecasting when there is any relationship between values especially in a time series in nature, the values before and the values after them. In this paper, we examine the application of goal programming as a mathematical tool for estimating the parameters of time series forecasting models such as the estimation of ARIMA model's parameters using traditional estimation and goal programming. Ordinary Least Squares (OLS) is used to estimate the ARIMA parameters using a linear constrained goal programming set. Maximum likelihood estimation and goal programming methods have been studied and compared using mean absolute error (MAE). We show that the GP prediction's mean absolute error values in the data set were significantly lower than those attained by the ARIMA model. These findings suggest that the prediction equations derived by goal programming were more accurate than those generated through maximum likelihood estimation. This can be formulated as minimizing the sum of absolute errors using goal programming as opposed to the ARIMA model's sum of squares error.

Keywords: *Goal Programming, Maximum Likelihood Estimation, Ordinary Least Squares*

INTRODUCTION

A strong mathematical technique for managing decision-making issues is linear programming, or LP. Many different optimization issues that arise in business, the military, government, and management have been solved by linear programming. Goal programming is another technique that has been effectively used to multi-criteria decision-making in several of the aforementioned disciplines. Goal programming was first proposed by Charnes and Cooper (1961), which Ghahtarani and Najafi (2013) mention as a method for reducing the total of all absolute deviations from the goals. Time series models have remained one of the most successful forecasting techniques in recent years for a wide range of business decisions. These applications include wind, weather, sales, and aviation forecasts, among many others. For seasonal forecasting and planning using meteorological and aviation data, Innis (2006) developed a seasonal clustering strategy that integrated statistical and mathematical programming methods. The performance accuracy of the Holt-Winters (HW), autoregressive integrated moving average (ARIMA), and exponential smoothing (ES) models for real oil price data is compared by Tularam & Saeed (2016). The ARIMA (2, 1, 2) model produced superior results than both HW and ES in six distinct areas.

In time series-based analysis, historical data on the item to be predicted is collected and analyzed to create a model that will represent the data generating process at its core and be used to predict future events.

There are two main methodologies that are commonly employed in time series forecasting, depending on the theory or assumption about the connection in the data sets (Box *et al*, 1994). The linear models that supports the traditional forecasting models—autoregressive (AR), moving average (MA), exponential smoothing (ES), Holt-Winters (HW), and autoregressive integrated moving average (ARIMA)—assume that the value of a time series will increase linearly as a function of previous observations (Efuwape *et al*, 2020).

Mohammadi *et al.*(2006) presented the autoregressive moving average (ARMA) model, which uses goal programming methods for parameter estimation in river flow forecasting.

The simultaneous optimization of all objectives and the mathematical formulation of goals and a set of linear constraints are the two primary disadvantages of goal-based programming (Arikan & Gungor, 2001). Many applications of linear goal programming, which was first developed by Zhang and Liu (2016), have been documented in the field of operations research.

Ozcan et al. (2017) developed a goal programming model using real data at a large hydroelectric power plant in Turkey. The implemented approach achieved a 91% reduction in production stoppages caused by operator mistakes while accounting for worker performance and task demands.

Varli et al. (2017) aimed to ensure that research assistant supervisory appointments at Kirikkale University's Faculty of Engineering were made in the most appropriate way possible in the past. Goal programming was the method used. 74 research assistants were assigned to 741 tests. They also discussed the problem of staff scheduling for nurses employed by hospitals in the endocrine and internal medicine departments. The monthly schedule was developed with consideration for hospital rules as well as the nurses' requests for particular authorization. This time, the goal programming method was used. An increase in service quality is the anticipated result of the improvements and solutions.

Bedir (2018) suggested that a hydroelectric power plant's production downtime cost might be decreased by employing the 0-1 Priority goal programming approach while accounting for personnel skills. It is possible to arrange competences using the PROMETHEE method. The factors affecting personnel competencies were weighted using the AHP approach. When the plant was operating at maximum efficiency in August 2017, the solution led to an 86% improvement. The current study, which focused on the staff competency rating in our survey, is comparable to research that has been done in the literature.

Gur and Eren (2018) used the goal programming approach to study scheduling and planning challenges. After the inspection, they conducted a thorough study and classified the issues. The use of goal programming as a technique for determining time series forecasting model parameters is examined in this work.

MATERIALS AND METHODS

Recall that for the OLS estimation,

$$y_t = X_t'\beta + \varepsilon_t, t = 1, \dots, T \qquad (1)$$

With the assumptions

1. $E(\varepsilon_t|X_t) = 0$
2. $E(X_tX_t')$ is non singular
3. $(y_t, X_t')'$ is stationary and weakly dependent

$\hat{\beta} \xrightarrow{p} \beta$ as $T \to \infty$

For the ARIMA model, that is the first order process, which means that the current values is based on the immediate preceding value.

$$y_t = \phi y_{t-1} + \varepsilon_t,\ t = 1, \dots, T \quad (2)$$

$\varepsilon_t \sim i.i.d\ N(0, \sigma^2)$ y_0 fixed / known

Equivalently, between (1) and (2), we have

$$X_t = y_{t-1}, \beta = \phi \quad (3)$$

Hence,

$$\hat{\phi}_{OLS} = \frac{\frac{1}{T}\sum_{t=1}^{T} y_{t-1} y_t}{\frac{1}{T}\sum_{t=1}^{T} y_{t-1}^2} \quad (4)$$

Recall that,

$$E(y_{t-1}^2) = \frac{\sigma^2}{1-\phi^2} \quad \text{if} \quad |\phi| < 1. \quad (5)$$

Since $|\phi| < 1$, (y_t) is stationary and weakly dependent.

If $|\phi| < 1$, then $\hat{\phi}_{OLS} \xrightarrow{p} \phi$

In estimating AR (1) parameter using goal programming.

$$\text{From } y_t = \phi y_{t-1} + \varepsilon_t \quad (6)$$

$$\varepsilon_t = y_t - \phi y_{t-1} \quad (7)$$

The general goal programming model can be expressed as follows.

Minimize:

$$min \sum_{i=1}^{NT} U_m \times d_i^- + V_m \times d_i^+ \ m = 1, \dots, 1 \quad (8)$$

Subject to the linear constraints:

Goal constraints:

$$\sum_{j=1}^{n} a_{ij} x_j + d_i^- - d_i^+, i = 1,2, \dots, m$$

System constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{bmatrix} \le \\ = \\ \ge \end{bmatrix} b_i, i = m+1, \dots, m+p$$

With $x_j, d_i^-, d_i^+ \ge 0$ for $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$

Equivalently, $d_i^- - d_i^+ = \varepsilon_t$ and $y_t = b$

$AR(1)$ can be written in terms of goal programming, we have;
Minimize

$$Z = \sum_{i=1}^{m} (d_i^- + d_i^+)$$

Subject to the linear constraints:
Goal constraints:

$$\phi y_{t-1} + \varepsilon_t$$

System constraints:

$$\phi y_{t-1} \begin{bmatrix} \le \\ = \\ \ge \end{bmatrix} y_t,$$

Note that, we have only one goal

RESULTS AND DISCUSSION

Table 1 depicts the mean and median coincide with positive value; this suggests a symmetrical distribution (bell-shaped).

Table 1. Summary Statistics for Inflation rates data

Variables	Inflation Rates
N	219
Mean	11.962
Standard Error Mean	0.221
Standard Deviation	3.274
Minimum	5.4
Q1	9.75
Median	11.9
Q3	13.7
Maximum	19.4
Skewness	0.15
Kurtosis	-0.43

Source: Author's Computations

According to Efuwape et al. (2020), skewness is a measure of asymmetry of a distribution, which can be positive or negative, or even undefined depending on the biasness of the tails. The skewness of inflation rate, which is 0.15 indicate that the data is skewed to the right. In addition, the Kurtosis describes the degree of how tall and sharp the central peak is, relative to a normal distribution. The result of kurtosis on inflation rate shows a value below 3.0, which means that they have normal frequency (platykurtic). This means that their central peak is lower and broader.

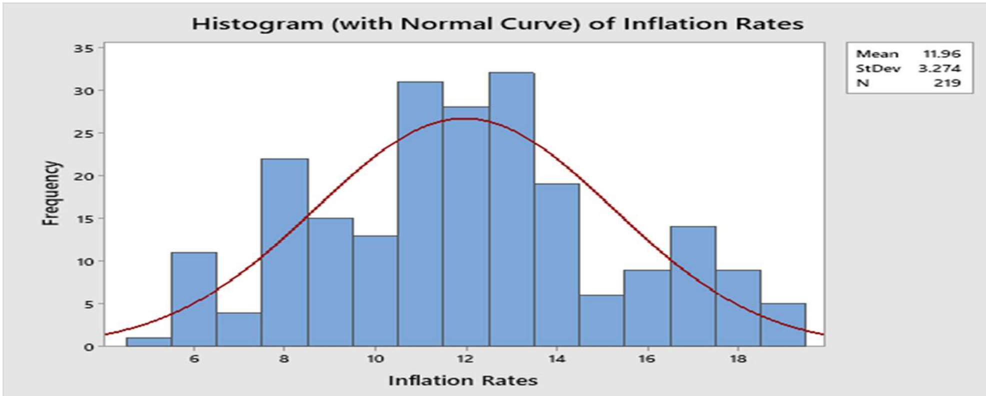


Figure 1. Histogram of Inflation rates

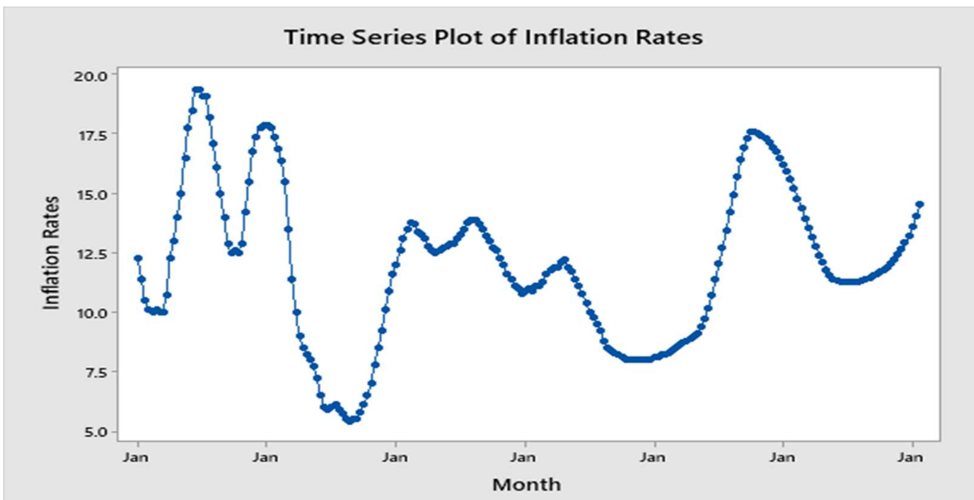


Figure 2. Time plot of inflation rates in Nigeria (January 2003- 2021)

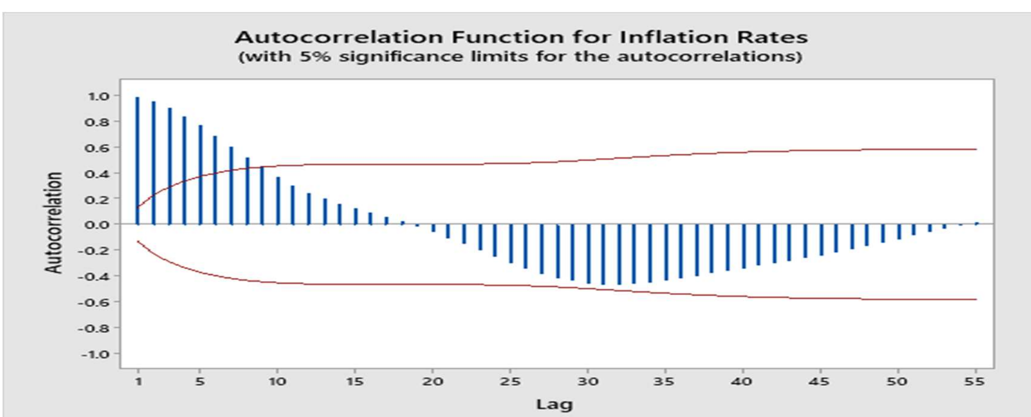


Figure 3. Autocorrelation function of inflation rate

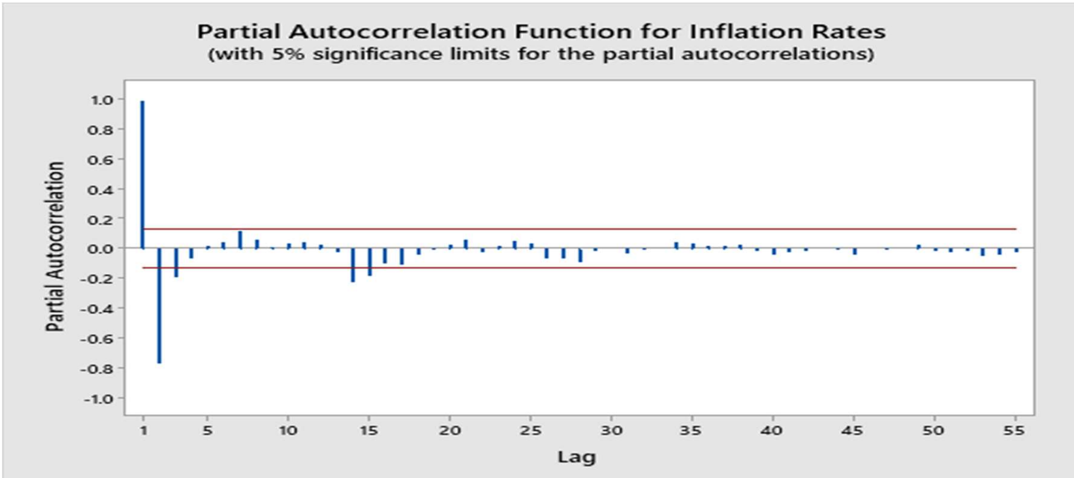


Figure 4. Partial Autocorrelation function of inflation rate

The short-term movement of the value in the series in various directions over the relevant period is depicted in Figure 2. Secular variation or secular movement refers to a sinusoidal rise in the inflation rate values over time. This plotted point generates a line by fitting a free-hand straight line onto the plotted points on the time plot for inflation rate stretching over the period, and this represents the trend of the time plot for inflation rate.

The result of ARIMA model parameters computed using maximum likelihood were then refined using an optimization method. Goal programming (GP) is a method that allows multiple objectives or goals to be attained simultaneously. In this method, the deviation from the goal is measured and after representing the objective function mathematically, a solution which minimizes the sum of the goal deviations is searched. Because of the ACF and PACF results, a maximum of 2 AR and MA parameters have been considered for analysis and all combinations of model parameters from ARIMA (1, 0, I) to ARIMA (2,0, 2) were tested. The proposed objective function is as follows:

$$min \sum_{i=1}^{NT} U_m \times EP_i + V_m \times EN_j \; m = 1, \dots, 12 \quad (9)$$

And constraints are:

$$AR_1 X_{t-1} + AR_2 X_{t-2} + \cdots + AR_p X_{t-p} + MA_1 R_{t-1} + MA_2 R_{t-2} + \cdots + MA_q R_{t-q} + C_m + EP_i - EN_j < (1 + div) X_i \quad (14)$$

$$0 \le EP_i \le Ediv X_i \; ; \quad 0 \le |EN_j| \le Ediv X_j$$

Autoregressive Integrated Moving Average Models

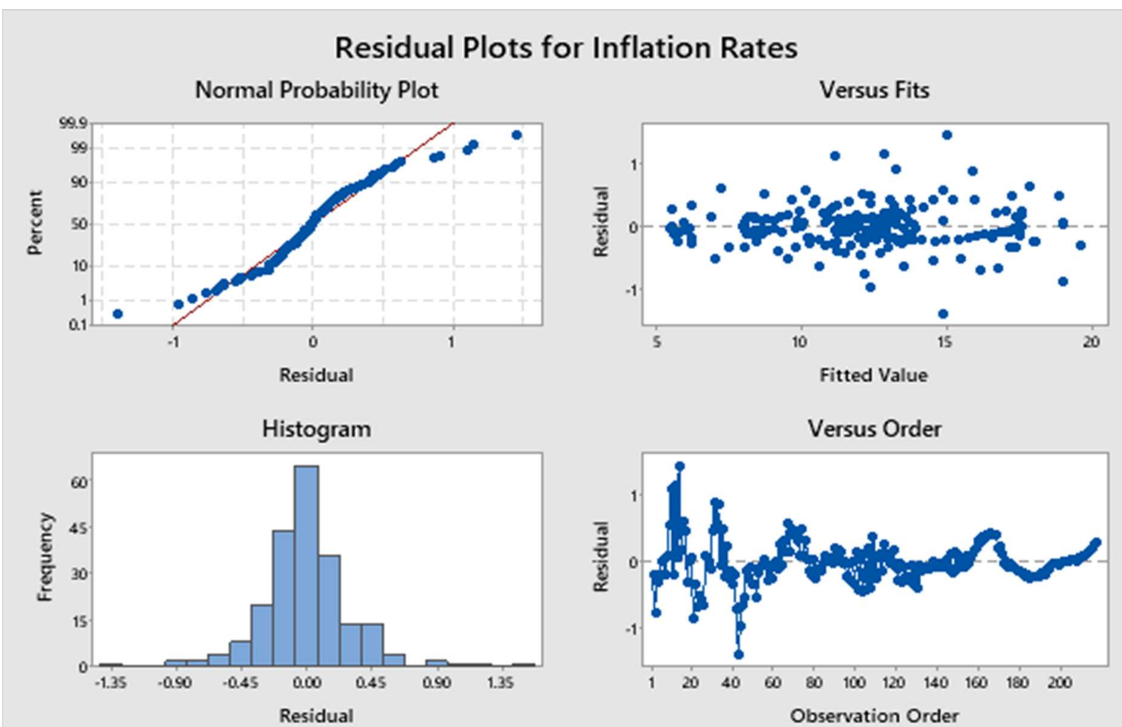
Tables 2 indicate the results of the models that relatively in agreement with the maximum likelihood method. Results show that when the number of AR and MA parameters increase, the GP method has a better performance compared with the maximum likelihood method. ARIMA (2, 0, 2) had better predictive capabilities. The mean absolute error was used as the defining criterion in comparing the model.

CONCLUSION

The data were modeled using the autoregressive integrated moving average. The maximum likelihood estimation and goal programming methods were investigated and compared, and the two techniques were compared using the mean absolute error (MAE). Goal programming was used to calculate the coefficients of the ARIMA model.

Table 2. Parameter estimation results for the complete time series

Model	Method	Mean absolute error			SD
		Maximum	Average	Minimum	
Without constant offset					
ARIMA (1, 0, 0)	Maximum likelihood	78.08	24.94	0.91	**18.71**
	GP	153.44	47.04	0.86	**35.58**
ARIMA (2, 0, 0)	Maximum likelihood	75.66	21.89	0.30	**18.26**
	GP	110.83	44.55	1.35	**27.81**
With constant offset					
ARIMA (1, 1, 0)	Maximum likelihood	70.76	26.95	0.23	**19.61**
	GP	87.57	28.21	0.17	**19.86**
ARIMA (2, 2, 0)	Maximum likelihood	67.25	20.43	0.06	**15.01**
	GP	104.36	31.46	0.39	**19.35**
ARIMA (1, 0, 1)	Maximum likelihood	79.77	19.27	1.03	**16.79**
	GP	90.16	19.48	0.25	**21.68**
ARIMA (2, 0, 2)	Maximum likelihood	72.3	21.87	0.26	**16.94**
	GP	**95.70**	**17.58**	**0.23**	**19.95**



The results for the inflation rates demonstrated that the procedure is effective and precise enough to achieve the intended goal. The goal programming method's biggest disadvantage is its high computing cost, particularly for models with several parameters. By utilizing effective optimization strategies to decrease the computing time and complexity, this unfavorable element can be substantially minimized. However, with the quick development of computer technology, this restriction should be significantly reduced, making it more practical to experiment with these new methods. Manual method was used to select the numbers of AR and MA parameters. For the optimization of these model parameters integer linear programming is suggested.

REFERENCES

Charnes A., and Cooper W.W (1961). Management models and the industrial application of linear programming. John wiley. Newyork vol. 1 and 2.

Ghahtarani, A., & Najafi, A. A. (2013). Robust goal programming for multi-objective portfolio selection problem. *Economic Modelling*, *33*, 588-592.

Inniss, T. R. (2006). Seasonal clustering technique for time series data. *European Journal of Operational Research*, *175*(1), 376-384.

Tularam Gurudeo Anand and Saeed Tareq (2016). Oil-price forecasting based on various univariate time-series models. *Americal journal of operation research*, 6(3): 226-235

Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (1994), Time Series Analysis: Forecasting and Control, 3rd Ed., Prentice-Hall, Inc., New Jersey.

Efuwape, B. T., Abdullah, K-K. A., Hammed, F. A., Obadina, O. G. and Awosanya, A. (2020). Estimating Autoregressive Model using Goal Programming. *Annals. Computer Series, Tibiscus University of Timisoara,* 18(2), 177-182. ISSN 1583-1765, www.anale-Informatica.ro.

Mohammadi, Eslami and Kahawita (2006) introduced an autoregressive moving average (ARMA) model for river flow forecasting using a goal programming methodology for parameter estimation.

Arıkan, F., & Güngör, Z. (2001). An application of fuzzy goal programming to a multiobjective project network problem. *Fuzzy sets and systems*, *119*(1), 49-58.

Zhang J., and Z. Liu (2016). Calculating some inverse linear programming problems. *Journal of Computational Applied Mathematics*, 72(2), pp. 261 – 273.

Özcan, E.C.; Varli, E.; Eren, T. (2017). Goal Programming Approach for Shift Scheduling Problems in Hydroelectric Power Plants. *J. Inf. Technol*., 10, 363–370.

Varli, E.; Ergis¸i, B.; Eren, T. (2017). Nurse Scheduling Problem with Special Constraints: Goal Programming Approach. *Erciyes Univ. J. Fac. Econ. Adm. Sci*., 49, 189–206.

Bedir, N. (2018). Shift Scheduling Problems Solution with Combined Ahppromethee and Goal Programming Methods: An Aplication in Hydroelectric Power Plant. Master 's Thesis, Kirikkale University, Kirikkale, Turkey.

Gür, S¸.; Eren, T. (2018). Scheduling and Planning in Service Systems with Goal Programming: Literature Review. *Mathematics*, 6, 265.